In the three lessons of this unit, you will learn statistical methods to help you make those decisions.

### Teenage Boys in the United States

<table>
<thead>
<tr>
<th></th>
<th>Smoke Cigarettes</th>
<th>Do Not Smoke Cigarettes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Get Lung Cancer</td>
<td>283,000</td>
<td>88,000</td>
</tr>
<tr>
<td>Do Not Get Lung</td>
<td>1,362,000</td>
<td>6,660,000</td>
</tr>
<tr>
<td>Cancer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,645,000</td>
<td>6,748,000</td>
</tr>
</tbody>
</table>
THINK ABOUT THIS SITUATION

Use the data on the previous page to help you think about the question of whether or not smoking causes lung cancer.

a. Is a smoker or a non-smoker more likely to get lung cancer? Explain your reasoning using the data reported in the table.

b. Is a boy who gets lung cancer more likely to be a smoker than a boy who does not get lung cancer? Explain your reasoning.

c. Based on the reported data, complete this sentence: A male smoker is ________ times as likely to get lung cancer as is a male non-smoker.

d. How many cases of lung cancer could be prevented if none of these boys smoked?

e. What exactly do people mean when they say that smoking causes lung cancer? Use the information in the table to support their position. Are there any plausible reasons other than smoking why smokers get lung cancer at a greater rate than non-smokers do?

f. Have you ever heard someone doubt that smoking causes lung cancer because he or she knew a smoker who had not gotten lung cancer? Or perhaps they knew someone who did have lung cancer and did not smoke. Do you think observations like these help to answer the question of whether smoking causes lung cancer? Explain your thinking.

Soon you must be prepared to make your own medical decisions and monitor your own medical care. The situation described above is typical of those you should understand: How does the proportion of one group who have a condition compare to the proportion in another group who have the condition?

INVESTIGATION 1

Summarizing and Displaying the Risk

The table on the previous page shows categorical data—the data collected are whether each boy falls into the category of smoker or non-smoker and into the category of lung cancer or not. People often find such a table difficult to interpret. Summary statistics and a thoughtfully made graph can make comparisons easier.

As you work on the problems in this investigation, look for answers to this question: What summary statistics and graphs can make categorical data easier to comprehend?

1 Absolute risk is defined as the proportion or percentage of people in a group for whom an undesirable event occurs. In college classrooms, students typically can choose their own seats. Professors have noticed a difference in grades between students who choose to sit in the front and those who choose to sit in the back. For example, in one math class, 9 of the 20 students who sat in the back failed the class, but only 3 of the 20 students who sat in the front failed the class. What was the absolute risk of failing the class for students who sat in the back? For students who sat in the front? Give your answers as fractions, proportions, and percents.
According to breastcancer.org, “Researchers estimate that 1 in 8 women will be diagnosed with invasive breast cancer at some time in their lives.”

What is the absolute risk, as a percent? Write your answer in a complete sentence, in context.

Medications to prevent loss of bone density may also suppress the body’s ability to replace older bone tissue with new bone tissue. Among 52,595 women who had been treated with such medications, an unusual type of fracture occurred in 117 of them. What is the absolute risk, as a percent? Write your answer in a complete sentence, in context. (Source: L. Park-Wylie, et al. “Bisphosphonate Use and the Risk of Subtrochanteric or Femoral Shaft Fractures in Older Women,” Journal of the American Medical Association, Vol. 305, 2011, pp. 783–89.)

Many teens have been vaccinated against the human papillomavirus (HPV), which can cause cancer in both men and women. According to the U.S. Centers for Disease Control and Prevention (CDC), this is a safe vaccine. Approximately 56 million doses have been distributed, and the CDC has received 21,194 reports of adverse events in females who received an HPV vaccine. Of these, 92.1% were classified as non-serious events such as fainting, nausea, or injection-site pain. The adverse events may or may not have been caused by the vaccine. (Source: www.cdc.gov/mmwr/preview/mmwrhtml/mm6229a4.htm)

a. What is the absolute risk of having an adverse event (whether caused by the vaccine or not), that is reported after receiving a dose of the HPV vaccine as a proportion? As a percent?

b. What is the absolute risk of a serious adverse event that is reported as a proportion? As a percent?

To display categorical data, you can create different types of bar graphs.

Examine the three different types of bar graphs on the facing page that display the data on male teenage smoking and lung cancer provided on page 2. For these data, the two groups are the boys who smoke cigarettes and the boys who do not. Whether a boy smokes is called the explanatory variable. The response variable is whether the boy eventually gets lung cancer.
When answering the questions at the top of page 6, be prepared to explain your thinking.
a. In the **stacked bar graphs** (sometimes called **segmented** bar graphs), are different bars defined by the explanatory variable or by the response variable? Is each bar segmented according to the explanatory variable or the response variable?

b. For each bar graph, describe the meaning of the leftmost bar.

c. Which graph shows most clearly which group has the larger absolute risk of getting lung cancer?

d. Which graph shows most clearly that only a small percentage of high school boys smoke cigarettes?

e. Which graph makes it easiest to compare the number of boys in the two groups who do not get lung cancer?

---

The best of intentions sometimes results in unanticipated consequences. A program in the Junnar region of India relocated leopards away from human-dominated areas. The leopards were moved into forests, an average of 24 miles away. In the three years before the program began, there were 12 leopard attacks on humans, 2 of them lethal and 10 not lethal. In the first three years after the program began, there were 50 attacks, 18 of them lethal and 32 not lethal.

Clearly, there were more attacks after the relocation program, but did the seriousness of the attacks also change? *(Source: Vidya Athreya et al. “Translocation as a Tool for Mitigating Conflict with Leopards in Human-Dominated Landscapes of India,” Conservation Biology, Vol. 25, 2011, pp. 133–141)*

a. To answer the question, what two groups must be compared? What is the response variable?

b. The information may be summarized in a **two-way frequency table**, such as that below. When making such a table, place the numbers for each group (explanatory variable) in different columns and place the levels of the response variable in different rows.

<table>
<thead>
<tr>
<th>Leopard Attacks on Humans</th>
<th>Before Program</th>
<th>After Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lethal</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>Not Lethal</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>50</td>
</tr>
</tbody>
</table>

Use the table to determine whether the absolute risk that an attack results in a death was higher before or after the relocation program. Explain your reasoning.
c. Use the data in the table to make a stacked bar graph (frequency on the vertical axis), a stacked bar graph (percent on the vertical axis), and a grouped bar graph (frequency on the vertical axis).

d. For each bar graph, write a sentence that summarizes information shown best by that graph.

7 To investigate whether middle school students think that being overweight is under a child’s control, students in 7th and 8th grade were given a questionnaire. One question asked was, “Is it the child’s fault if they are fat?” Of the 84 boys who answered the question, 25 said yes, 24 said no, and 35 said they were not sure. Of the 138 girls, 23 said yes, 61 said no, and 54 said they were not sure. (Source: Paul B. Rukavina and Weidong Li, “Adolescents’ Perceptions of Controllability and Its Relationship to Explicit Obesity Bias,” Journal of School Health, Vol. 81, January 2011, pp. 8–14)

a. What two groups were compared? What is the response variable?
b. Summarize the information in a two-way frequency table.
c. Compare two values to answer this question: Was a boy or a girl more likely to say yes?
d. Make the bar graph that best compares the proportions of boys and girls who gave each answer.
e. What can you conclude about any difference in how boys and girls respond to this question?

8 Sometimes you will see data such as that in Problem 7 displayed by a pie chart. In the pie chart below, the central angle for each of the three sectors is proportional to the percentage of boys who gave the indicated answer.

<table>
<thead>
<tr>
<th>Boys’ Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Sure</td>
</tr>
<tr>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
</tr>
</tbody>
</table>

a. Verify, by calculation, that the measure of the central angle for the “not sure” answer should be 150°.
b. Compute the measure of the central angle for the “yes” answer. For the “no” answer.

c. Compute the measure of the central angle for each of the girls’ answers and make a pie chart.

d. Most statisticians recommend including stacked bar graphs rather than pie charts in reports, even if you have technology that easily makes both. Give a reason why you think stacked bar graphs are better than pie charts.

**SUMMARIZE THE MATHEMATICS**

In this investigation, you learned to use summary statistics and graphs to compare groups sorted by the same categorical variable.

a. What is absolute risk?

b. When comparing situations involving risk, how can you tell which is the explanatory variable and which is the response variable?

c. Describe three types of bar graphs and the characteristics of the data that are best shown by each one.

d. What steps are involved in making a pie chart?

*Be prepared to share your ideas and reasoning with the class.*

**CHECK YOUR UNDERSTANDING**

A “food-secure” household is one that has access at all times to enough food for an active healthy life for all household members. In 2009 in the United States, 194,579,000 adults and 57,010,000 children lived in food-secure households; 20,741,000 adults and 16,209,000 children lived in households with low food security; and 12,223,000 adults and 988,000 children lived in households with very low food security. ([Source: U.S. Census Bureau, Statistical Abstract of the United States: 2012, Table 214.](https://www.census.gov/programs-surveys/sah/2012-214.html))

a. Summarize the information in a two-way frequency table with adult/child as the explanatory variable.

b. In 2009, was an adult or a child more likely to live in a household with very low food security? What two values do you need to compare to answer this question?

c. Make a bar graph that best compares the proportions of adults and children who live in households with the three levels of food security. Describe what you can conclude from this graph.
Comparing Risk

In this investigation, you will examine how reports in the media use percentages to compare the prevalence of some condition in two different groups.

As you work on the following problems, look for answers to this question:

How is the risk associated with two different conditions best compared?

1 In a study of over 15,000 teens, 62% of boys and 55% of girls consumed sugar-sweetened soda on the previous day. (Source: Nalini Ranjit et al. “Dietary and Activity Correlates of Sugar-Sweetened Beverage Consumption Among Adolescents.” Pediatrics published online Sep. 27, 2010.) When asked to compare the boys’ percentage with the girls’ percentage, two students gave the answers below. Show how each student computed the percentage.

Alma: The boys’ percentage is 7% bigger than the girls’.

Bill: The boys’ percentage is about 113% of the girls’.

2 Melanoma is the most dangerous type of skin cancer. The International Agency for Research on Cancer (IARC) reported that the absolute risk of melanoma over eight years for women who do not use tanning beds regularly is only about two-tenths of 1 percent. However, the risk of melanoma is increased by 75% when the use of tanning beds starts before age 30. That sounds serious, but what, exactly, do they mean? (Sources: Fatiha El Ghissassi et al. “A Review of Human Carcinogens—Part D: Radiation,” The Lancet Oncology, Vol. 10, August 2009, pp. 751–752, www.delawareonline.com/article/20100304/NEWS/105070009, www.iarc.fr/en/publications/index.php)

a. What could be the absolute risk of melanoma for those who do use tanning beds regularly? Give two answers to this question, based on the thinking of Alma and Bill in Problem 1.

b. Which answer from Part a must be the absolute risk of melanoma for those who tan regularly? Why?

To try to prevent confusion, medical professionals typically give one of two standard statistics when helping patients compare risk:

absolute risk reduction = absolute risk for one group − absolute risk for another group

relative risk = absolute risk for one group
                           absolute risk for another group

 Typically, the proportion for the group with the larger absolute risk is placed so that the absolute risk reduction is positive and the relative risk is greater than 1.
Refer back to Problem 1 involving the consumption of sugar-sweetened soda.

a. Which student computed absolute risk reduction? Did either student compute something like relative risk?

b. Read the descriptions below about Problem 1. Which statement is the clearest interpretation of absolute risk reduction? Which statement is the clearest interpretation of relative risk? Be prepared to explain your reasoning.
   
   I. The boys were 7% more likely than the girls to consume sugar-sweetened soda on the previous day.

   II. The difference in the percentage of the boys and the percentage of girls who consumed sugar-sweetened soda on the previous day is 7 percentage points.

   III. A boy was about 1.13 times more likely than a girl to have consumed sugar-sweetened soda on the previous day.

   IV. The boys were about 113% more likely than the girls to consume sugar-sweetened soda on the previous day.

Now refer to the information on the use of tanning beds and the development of melanoma in Problem 2.

a. What is the absolute risk reduction in melanoma from regular tanning versus no regular tanning, expressed in percentage points? Use your answer in a sentence that describes its meaning.

b. What is the relative risk of melanoma from regular tanning versus no regular tanning? Use your answer in a sentence that describes its meaning.

c. The units for absolute risk reduction usually are percentage points. What can you say about the units for relative risk?

d. In Part a, the absolute risk reduction may seem very small. However, it has been estimated that every year nearly 2.3 million American teens use tanning beds. How many cases of melanoma do you estimate could be prevented if they did not use tanning beds?
Baseball pitchers aged 9 to 14 were followed for 10 years. Four out of the 26 who had pitched at least 100 innings in one year had elbow surgery, shoulder surgery, or retirement due to a throwing injury. Of 128 who had not pitched this much, 6 had such problems. (Source: G. Fleisig, et al. "Risk of Serious Injury for Young Baseball Pitchers: A 10-Year Prospective Study," American Journal of Sports Medicine, Vol. 39, February 2011, pp. 253–257)

a. Compute the percentage of each group who have such problems. Compute the difference of the percentages. What is the name for this difference? Write an explanation, for parents of young pitchers, that explains its meaning.

b. Compute the ratio of the percentages of each group who have such problems. What is the name for this difference? Then write an explanation, for parents of young pitchers, that explains its meaning.

Asthma is a chronic disease in which there are episodes of shortness of breath and wheezing due to inflammation and narrowing of small airways in the lungs. About 9.6% of teens and children under age 18 in the United States have asthma. One of the most-commonly prescribed medicines for asthma is Advair. Like all medications, Advair can have side effects—undesirable symptoms brought on by the medicine. A 28-week clinical trial was conducted to determine the side effects of one of the components of Advair, salmeterol. Available patients age 12 to 18 were randomly divided into two groups. One group of 1,653 patients received salmeterol. The other group of 1,622 patients received a placebo, which had no active ingredient in it. At the end of the study, researchers counted how many patients in each group had been hospitalized for any cause.

The sentence below summarizes the results of the Advair clinical trial.

Hospitalization from all causes was increased in the salmeterol group \((2\%) \left(\frac{35}{1,653}\right)\) vs. the placebo group \((<1\%) \left(\frac{16}{1,622}\right)\) [relative risk 2.1].

(Sources: National Health Statistics Reports, Number 32, January 12, 2011; Advair Medication Guide, GlaxoSmithKline, 2010)

a. What is the meaning of \(\frac{35}{1,653}\) and \(\frac{16}{1,622}\)?

b. Show how the 2% and <1% summaries were computed.

c. Verify the computation of the relative risk.

d. Compute the absolute risk reduction.

e. Write a paragraph for patients explaining the risk.
Immunizations are not equally effective overall and are not effective for all people. For example, the polio vaccine is about 99% effective in children who receive the full series. That means that for every 100 immunized children who are exposed to polio, only 1 is expected to get the disease. In contrast, the influenza vaccine is only about 66% effective. *(Source: www.cdc.gov)*

Here are three hypothetical diseases.

**Disease A:** The absolute risk of getting disease A for those who have been immunized is 1%. The absolute risk for those who have not been immunized is 2%.

**Disease B:** The absolute risk of getting disease B for those who have been immunized is 34%. The absolute risk for those who have not been immunized is 68%.

**Disease C:** The absolute risk of getting disease C for those who have been immunized is 98%. The absolute risk for those who have not been immunized is 99%.

a. For each disease, compute the absolute risk reduction for those who have been immunized.

b. Identify the two diseases with the same absolute risk reduction. Suppose that both diseases are similarly serious but you have the resources to provide vaccinations for only one of them. For which disease would you choose to vaccinate, or does it matter? Why?

c. For each disease, compute the relative risk.

d. Identify the two diseases with the same relative risk. Suppose that both diseases are similarly serious but you have the resources to provide vaccinations for only one of them. For which disease would you choose to vaccinate, or does it matter? Explain your reasoning.

**SUMMARIZE THE MATHEMATICS**

In this investigation, you learned two methods of comparing risk.

a. What is absolute risk reduction? Give an example of how it is interpreted.

b. What is relative risk? Give an example of how it is interpreted.

c. Sometimes a condition is extremely serious, but quite rare, such as melanoma. In such a case, discuss whether it is better to tell people about the absolute risk reduction from not engaging in the risky behavior, or tell them about the relative risk.

*Be prepared to explain your ideas and reasoning to the class.*
Measles is a highly contagious childhood disease that can cause complications including death. There was an outbreak of measles in Colorado in December 1994. Out of 609 children who had been vaccinated, 10 got measles (and those 10 children had been given only one of the two recommended doses). Of the 16 children who were not vaccinated, 7 got measles. (Source: C. R. Vitek et al. “Increased Protections During a Measles Outbreak of Children Previously Vaccinated with a Second Dose of Measles-Mumps-Rubella Vaccine,” Pediatric Infectious Diseases Journal, Vol. 18, July 1999, pp. 620–623.)

a. What is the explanatory variable? What is the response variable?

b. Make a two-way frequency table that summarizes this information.

c. For each group, compute the absolute risk of getting measles.

d. Compute the absolute risk reduction.

e. Compute the relative risk of getting measles.

f. Write a paragraph for parents using your results in Parts c, d, and e.

Design of Experiments

As you may have discussed in the Think About This Situation (page 3), it is impossible to prove that smoking causes lung cancer just by collecting data that show that smokers are more likely than non-smokers to get lung cancer. Such an observational study leaves open the possibility that the people who smoke tend to be the same people who do something else that is actually causing the increased rate of lung cancer.

As you work on the problems in this investigation, look for answers to this question:

How can you design an experiment that provides convincing evidence that one treatment causes a different response than another treatment?
Nosebleeds are common, and you may have heard one of many folk remedies about how to stop them. One such suggestion is to drop car keys down the back of the neck, inside the shirt. People have said things like, “I tried the car-key trick, and in no time at all, my nosebleed stopped.” (Source: People’s Pharmacy, Los Angeles Times, February 28, 2011.)

a. Evidence such as this (“it worked for me”) is called **anecdotal evidence**. Why is anecdotal evidence not convincing that the car-key trick causes nosebleeds to stop?

b. Think about what is meant when a person says that a certain remedy causes nosebleeds to stop. Which statement below best conveys the intent of the person?

I. The person wants you to believe that it works in every case.

II. The person wants you to believe that a nosebleed will not stop unless this remedy is used.

III. The person wants you to believe that, for some people, nosebleeds usually stop sooner if they use the remedy than if they do not use the remedy.

Few treatments work in all cases. Even vaccines, as you saw in the previous investigation, are not effective for all people. To further complicate matters, not everyone who is exposed to a disease gets the disease even if he or she is unvaccinated. So, we say that a vaccine “works” when exposed people are less likely to get a disease if they are immunized than if they are not immunized. To show that this is the case, scientists must conduct an *experiment*.

In a typical experiment, two or more treatments (conditions you want to compare) are randomly assigned to an available group of people (or animals, plants, or products), called subjects. The purpose of an experiment is to establish cause and effect—does one treatment cause a different response than the other treatment? A well-designed experiment must have three characteristics:

- **Random assignment**: Treatments are assigned randomly to the subjects.

- **Comparison group or control group**: At least two groups get different treatments and then are compared. Alternatively, one group gets the treatment under study and then is compared to another group that does not get a real treatment (a control group).

- **Sufficient number of subjects**: Subjects will vary in their responses, even when they are treated alike. If there are not enough subjects, this variability of responses within each treatment group may obscure any difference between the effects of the treatments. Deciding how many subjects are sufficient is one of the more difficult tasks that statisticians perform.

After treatments are completed, the response of each subject is determined. Then the responses for each treatment are summarized and compared.
Injuries of the leg, knee, and foot are common in people who do a lot of running, such as military recruits undergoing basic training. Researchers wanted to know whether foot orthotics—shoe inserts custom-molded to the person’s foot—might help prevent such injuries.

Four hundred military officer trainees were identified as medium or high risk for leg, knee, or foot injury. They were randomly split into two groups. One group got custom orthotics and the other did not. After seven weeks of basic training, 21 of the 200 who had received custom orthotics had a leg, knee, or foot injury that required that he or she stop physical training for two or more days. Sixty-one of the 200 who did not get orthotics had such injuries.


a. Who are the subjects in this experiment? What are the treatments? What is the response variable?

b. Does this experiment have the first two characteristics of a well-designed experiment? Explain.

c. What was the absolute risk of leg, knee, or foot injury for each group?

d. The researchers hypothesized that the injury rate would be lower among those who wear foot orthotics than among those who do not. Do you think that their experiment provides strong, weak, or no evidence for that hypothesis? Explain.

Many studies have shown that people tend to do better when they are given special attention or when they believe they are getting competent medical care. This is called the placebo effect. Even people with post-surgical pain report less discomfort if they are given a pill that they think is a painkiller but actually contains no medicine. One way to control for the placebo effect is to make the experiment single blind—the person receiving the treatment does not know which treatment he or she is getting. That is, subjects in all treatment groups appear to be treated exactly the same way. This often calls for the use of a placebo, a fake treatment that has no medical value, but looks like a real treatment to the person receiving it. In a double-blind experiment, neither the subject nor the people who administer the treatment and evaluate how well it works know which treatment the subject received.
3. What can you do if you have a toothache and cannot immediately get to the dentist? Some people recommend dabbing a small amount of clove oil near the tooth. (Clove oil is toxic, so only a very small amount must be used.) Does that work? Read the following report from the *New York Times*.

In a study published in *The Journal of Dentistry* in 2006, for example, a team of dentists recruited 73 adult volunteers and randomly split them into groups that had one of four substances applied to the gums just above the maxillary canine teeth: a clove gel, benzocaine, a placebo resembling the clove gel, or a placebo resembling benzocaine. Then, after five minutes, they compared what happened when the subjects received two needle sticks in those areas. Not surprisingly, the placebos failed to numb the tissue against the pain, but the clove and benzocaine applications numbed the tissue equally well.


a. In this experiment, what treatments were tested? What was the response variable?

b. Why was it important to split the volunteers randomly into the treatment groups rather than letting them decide which group they wanted to be in?

c. What were the placebos in this experiment?

d. This experiment was single blind. Why was this precaution necessary?

e. About how many adults would have been in each group? Could that be sufficient to convince you that the clove oil is as effective as the benzocaine? Create some hypothetical data from 73 adults in such an experiment that would be convincing.

4. A study of young baseball pitchers found that 7 of the 103 who had thrown curveballs before the age of 13 were injured seriously enough to have surgery or quit pitching. Of the 187 who had not thrown curveballs before age 13, 8 were injured seriously enough to have surgery or quit pitching. The researchers had hypothesized that throwing curveballs before age 13 put young pitchers at risk for injury. *(Source: G. Fleisig, et al. “Risk of Serious Injury for Young Baseball Pitchers: A 10-year Prospective Study,” American Journal of Sports Medicine, Vol. 39, February 2011, pp. 253–257.)*

a. Display the data in a two-way frequency table.
b. What two conditions are being compared in this study?

c. What is the response variable?

d. What is the absolute risk of injury for those pitchers who had thrown curveballs before age 13? For those who had not?

e. Would you say that the researchers have strong, weak, or no evidence for their hypothesis? Explain.

f. Does this study satisfy the first two characteristics of a well-designed experiment?

5 A lurking variable helps to explain the association between the treatments and the response, but is not the explanation that the study was designed to test. Treatments are assigned randomly to subjects to equalize the effects of possible lurking variables among the treatment groups as much as possible. Refer to the measles study in the Investigation 2 Check Your Understanding (page 13). Can you think of possible lurking variables that might explain why those who were not immunized were more likely to get measles than those who were immunized?

6 Assume that you are the nurse at a large elementary school where children frequently come to you with nosebleeds.

Design an experiment to test whether dropping keys down the back of the neck causes nosebleeds to stop. (Assume parents of all children in the school have given permission for their participation in the experiment and you have followed ethical guidelines for research with human subjects.)

One possible explanation for why keys down the back might actually be effective in stopping a nosebleed is that they are cold, causing blood vessels to constrict.

a. What are the treatments in your experiment? How might you use a placebo?

b. Describe the variable you will use as the response.

c. Treatments should be randomly assigned to the children. Describe how you could accomplish this. That is, when a child with a nosebleed comes to the nurse's office, how will you know which treatment he or she should get?

d. Write a protocol, or description of how to proceed, when a child with a nosebleed comes into the nurse's office.

e. Is your experiment single or double blind? Explain.
SUMMARIZE THE MATHEMATICS

In this investigation, you examined the characteristics of well-designed experiments.

a. What are the three characteristics of a well-designed experiment? Why is each necessary?

b. Why are single blinding and double blinding desirable in an experiment?

c. What is the placebo effect? How can you account for it when designing an experiment?

Be prepared to share your ideas and reasoning with the class.

CHECK YOUR UNDERSTANDING

Parkinson’s disease is a progressive disease that results in loss of motor function, among other problems. Currently, there is no cure and only partial treatment.

In a clinical trial to test a new gene therapy, 45 patients aged 30 to 75 years with typical, advanced Parkinson’s disease had tubes implanted in their brains. Twenty-two were randomly selected to receive infusions through the tube of a saline solution containing a gene that should increase levels of a brain chemical that is missing in people with Parkinson’s. The other 23 patients received only saline solution through the tube. Neither the patients nor the specialist who evaluated how well they were doing knew which treatment the patient received. After six months, the patients who received the gene therapy had a 23.1% improvement in their score on a test of motor ability (which included assessment of such things as problems walking, problems with speech, and severity of muscle tremors). The patients who received the sham treatment had a 12.7% improvement.


a. Who are the subjects in this study? What are the treatments?

b. Describe the response variable.

c. Does this study have the first two characteristics of a well-designed experiment? Explain.

d. Is this study single blind? Is it double blind? Explain.
ON YOUR OWN

These tasks provide opportunities for you to use and strengthen your understanding of the ideas you have learned in the lesson.

1 College students in Korea and in the American Midwest participated in a study. They were told to pretend that they were sick and that a meeting with classmates to work on a group project was scheduled for tomorrow. Each student then composed an email to the members of their group asking that the meeting be rescheduled. Out of 127 Koreans, 105 included apologies in their message, while 51 out of 97 Americans included apologies. (Americans tended, instead, to write things like “I would really appreciate” or “Thank you,” while Koreans rarely used such language.) (Source: Hye Eun Lee & Hee Sun Park, “Why Koreans Are More Likely to Favor ‘Apology,’ While Americans Are More Likely to Favor ‘Thank You,’” Human Communication Research, Vol. 37, 2011, pp. 125–146.)

a. What two groups are being compared? What is the response variable?
b. Summarize the information in a two-way frequency table.
c. Make a bar graph that best compares the proportion of each group that included an apology.
d. Based on your graph, write a summary sentence or two describing how the two groups differ.

2 Artificial turf on athletic fields was first introduced in the 1960s. Its safety has been controversial since then. One issue that has been investigated is whether injuries of football players tend to be more serious on artificial turf than on grass. A study followed 24 NCAA Division 1A college football teams over three seasons. In the games played on FieldTurf (an artificial turf), there were 1,050 injuries, 875 of which were minor, 114 were substantial, and 61 were severe. In about the same number of games played on grass, there were 1,203 injuries, 938 of which were minor, 169 were substantial, and 96 were severe. (Source: Michael C. Meyers, “Incidence, Mechanisms, and Severity of Game-Related College Football Injuries on FieldTurf Versus Natural Grass: A 3-Year Prospective Study,” American Journal of Sports Medicine, Vol. 38, 2010, pp. 687–697.)

a. What two groups are being compared? What is the response variable?
b. Summarize the information in a two-way frequency table.

c. Make a stacked bar graph (frequency on the vertical axis) and a stacked bar graph (percent on the vertical axis). Check your bar graph against the stacked bar graph produced using TCMS-Tools data analysis.

d. Write a paragraph to answer the question of whether football injuries during games played on artificial turf and on grass tend to differ in severity.

3 People have become increasingly concerned that loud music, especially that played through headphones and MP3 players, is damaging the hearing of teens. In part to test whether that might be the case, hearing loss in teens in 1988–1994 (before MP3 players were available) was compared to hearing loss in teens in 2005–2006. Of 796 teens aged 18 or 19 in 1988–1994, 15.2% had some hearing loss. Of 413 teens aged 18 or 19 in 2005–2006, 20.1% had some hearing loss.


a. What two groups are being compared? What is the response variable?

d. In Part c, the absolute risk reduction may seem fairly small. But suppose that a high school has 4,000 students, all of whom use headphones and MP3 players, and it is true that these cause hearing loss. How many cases of hearing loss do you estimate could be prevented if the students did not use them?

e. Compute and interpret the relative risk of hearing loss for the two groups.
In another study of the safety of tanning beds, 375 women who got melanoma before age 40 were compared to 275 similar women who had not. Of the women who got melanoma, 103 had used tanning beds. Of those who had not gotten melanoma, 67 had used tanning beds. (Source: Anne E. Cust et al. “Sunbed Use During Adolescence and Early Adulthood Is Associated with Increased Risk of Early-Onset Melanoma,” International Journal of Cancer, 2010.)

a. What two groups are being compared? What is the response variable?

b. Display the data in a two-way frequency table.

c. Was a woman who got melanoma more likely to have used a tanning bed than a woman who did not get melanoma? Explain your reasoning.

d. Complete this sentence: A woman who got melanoma before age 40 was _____ times more likely to have used a tanning bed than a woman who did not get melanoma.

Refer back to the information about the asthma medication study from Problem 6 in Investigation 2 (page 11).

a. Who are the subjects in this study? What are the treatments? What is the response variable?

b. Does this study have the first two characteristics of a well-designed experiment? Explain.

c. Is this study single blind? Is it double blind? Explain.

A group of college students wanted to see if other college students would obey instructions better when given by a professionally dressed person or a casually dressed person. Two videos of the same 22 year-old actor were made. She gave the same instructions in each, but she was wearing different clothing and hair style. Students were randomly assigned to watch one of the two videos. Of the 32 students who watched the professionally dressed version of the video, 41% correctly followed instructions to write their gender and grade level on a test they were given. Of the 35 students who watched the casually dressed video, 63% wrote this information. (Source: Anastacia E. Damon et al. “Dressed to Influence: The Effects of Experimenter Dress on Participant Compliance,” Undergraduate Research Journal for Human Sciences, Vol. 9, 2010, www.kon.org/urc/v9/damon.html)

a. What are the treatments in this study? What is the response variable?

b. Does this study have the first two characteristics of a well-designed experiment?

c. Is this study single blind? Explain.

d. Identify a lurking (hidden) variable that could account for the results.
ON YOUR OWN

7 Look back at your work on Applications Task 4. Would you say that the researchers have strong, weak, or no evidence that melanoma is associated with the use of tanning beds? Explain your reasoning.

CONNECTIONS

These tasks will help you build links between mathematics you have studied in the lesson and to connect those topics with other mathematics you know.

8 The recommended level of physical activity for a 12th-grade boy or girl is at least 60 minutes per day at least five days a week of activity that increases heart rate and makes him or her breathe hard. Of 12th-grade girls, 20.6% meet this recommendation, 43.2% exercise at least 60 minutes but for only one to four days a week, and 36.2% do not exercise at least 60 minutes on any day of the week. Of 12th-grade boys, the respective percentages are 38.7%, 39.8%, and 21.5%. (Source: U.S. Census Bureau, Statistical Abstract of the United States: 2011, Table 209.)

a. Construct pie charts for these data.

b. Construct a stacked bar graph (percent on the vertical axis).

c. Do boys or girls tend to do better meeting the recommendation? How much better? Which graphic better shows this?

9 Reports in the media often mention a percentage increase or percentage decrease. Carefully examine each of the following reports.

a. Some people think that rhino horns have medicinal uses, so rhinos are often killed illegally so their horns can be ground into powder. A UPI headline read, “Rhino poaching in South Africa increased by 50 percent in 2013.” The following article said that 1,004 rhinos were killed in South Africa in 2013, but did not give the number killed in 2012. (Source: www.upi.com/Science_News/Blog/2014/01/17/Rhino-poaching-in-South-Africa-increased-by-50-percent-in-2013/3981389975097/)

i. About how many rhinos were killed in South Africa in 2012?

ii. A CNN article published about the same time gave the number of rhinos killed in 2012 (the number you computed in part i) and the number of rhinos killed in 2013 (which is 1,004). It said that “The 2013 record is almost double the year before.” Is CNN correct? (Source: www.cnn.com/2014/01/18/world/africa/south-africa-record-rhinos-poached/)
b. The Measles & Rubella Initiative reported:

With intervention by the Measles & Rubella Initiative and commitment from governments around the world, global measles deaths worldwide fell by 74 percent between 2000 and 2010, from an estimated 535,000 to 139,300.

Source: www.measlesrubellainitiative.org

i. Show how the percentage decrease of 74% was computed.

ii. Is the 74% one of the statistics you studied in this lesson or something different? Explain.

iii. Assuming that the world population was equal in 2000 and 2010, show how you can compute the relative risk of dying from measles in those two years.

iv. Can you compute the absolute risk reduction? Explain.

c. The Cary News reported that 16,804 high school students dropped out of North Carolina’s public schools in 2009–2010, a 12.4 percent decrease over the year before. The newspaper said the dropout rate in 2009–2010 was 3.75% but was 4.27% in 2008–2009. (Source: www.carynews.com/2011/03/16/30046/schools-retain-more-students.html, March 16, 2011)

i. Compute the percentage decrease in number of high school dropouts based on the information given. Compare your answer with the reported answer.

ii. How many students were there in North Carolina’s public schools in 2009–10?

iii. Assuming that the total number of students was the same as in 2009–10, how many students dropped out in 2008–09?

iv. Compute the relative risk of dropping out for the two years. Then compute the absolute risk reduction.

d. A survey in 22 public high schools in Boston asked students about physical date violence. It found that physical date violence was associated with alcohol use. In a table giving the absolute risk of physical date violence perpetration by risk behaviors and sex, a relative risk of 2.05 was reported for boys who used alcohol in the past 30 days compared to those who had not. (Source: Emily F. Rothman et al. “Perpetration of Physical Assault Against Dating Partners, Peers, and Siblings Among a Locally Representative Sample of High School Students in Boston, Massachusetts,” Archives of Pediatric and Adolescent Medicine, Vol. 164, December 2010, pp. 1118–1124.) Explain the meaning of the relative risk of 2.05, in the context of this situation.
ON YOUR OWN

10 In a rural area of Bangladesh, more than 10% of children die by 10 years of age. A study followed 144,858 Bangladeshi newborns. Among the 1,385 children whose mothers had died, only 24% survived to their 10th birthday. Of the 143,473 whose mothers lived, 89% survived to their 10th birthday. (Source: Carine Ronsmans et al. “Effect of Parent’s Death on Child Survival in Rural Bangladesh: A Cohort Study,” The Lancet, Vol. 375, June 2010, pp. 2024–2031.)

   a. Make a two-way frequency table summarizing this situation.
   b. Overall, what percentage of children died by age 10?
   c. Of the children who died, what percentage had mothers who died?
   d. Write a sentence or two for a newspaper report giving the absolute risk reduction of death for children whose mothers remained alive compared to those whose mothers died.
   e. Write a sentence or two for a newspaper report giving the relative risk for children whose mothers died compared to those whose mothers remained alive.

11 Healthy adults need at least 7 hours of sleep a night. (Teens need at least 9 hours.) A survey of 74,571 Americans 18 and older found that 35.3% reported sleeping less than 7 hours per night on average. People who reported sleeping less than 7 hours were more likely to report unintentionally falling asleep during the day at least once in the preceding 30 days (46.2% versus 33.2%). They were more likely to report nodding off or falling asleep while driving in the preceding 30 days (7.3% versus 3.0%). They also were more likely to report snoring (51.4% versus 46.0%). (Source: CDC Morbidity and Mortality Weekly Report, March 4, 2011, www.cdc.gov/mmwr/preview/mmwrhtml/mm6008a2.htm?s_cid=mm6008a2_w)

   a. Make a two-way frequency table, with one column for those who get at least 7 hours of sleep and one for those who do not. The rows will represent the number in each group who did and did not report snoring.
   b. What percentage of all of the Americans surveyed snore?
   c. What is the relative risk of nodding off or falling asleep while driving in the preceding 30 days for the two groups? Use this ratio in a sentence that explains what it means.
d. What percentage of all of the Americans surveyed nodded off or fell asleep while driving in the preceding 30 days?

e. Would you say that the increased risk of nodding off or falling asleep while driving is higher or lower than the increased risk of snoring for those who do not get 7 hours of sleep on average? Explain how you decided this.

People like to speculate about “curses” attached to various events that are otherwise positive for the person involved. For example, “Researchers compared actresses who won Best Actress statuettes from 1936 to 2010 to those who were nominated but didn’t win, and found that winners were, indeed, 1.68 times as likely to divorce as non-winners. Of the 265 married nominees, 159 eventually divorced—a whopping 60 percent.” In the last sentence, “nominees” means all of the actresses nominated, both those who won the Oscar and those who did not.

Your goal in this task is to compute the proportion of winners who were divorced and the proportion of non-winners who were divorced. (Sources: H. Colleen Stuart, et al. “The Oscar Curse: Status Dynamics and Gender Differences in Marital Survival,” Social Sciences Research Network, January 27, 2011; www.huffingtonpost.com/2011/01/31/oscar-curse-study-research_n_816295.html?ir=Entertainment)

a. Let \( x \) be the number of winners who eventually divorced. Let \( y \) be the number of non-winners who eventually divorced. Write an equation that gives the sum of \( x \) and \( y \).

b. Typically, five women are nominated for Best Actress and one of them is the winner. What is a reasonable estimate of the number of the 265 married nominees who were winners? What is a reasonable estimate of the number who did not win?

c. Write the expression that was used to compute the relative risk of 1.68.

d. Parts a and c lead to two equations in two unknowns, say \( x \) and \( y \).

   i. Write and solve the system of equations.

   ii. Then, find the proportion of winners who were divorced.

   iii. Find the proportion of non-winners who were divorced.

e. Do you think it is better to report that winners were 1.68 times more likely to divorce or would something else have been more clear to most people? Explain.
According to an article on magicvalley.com, “One in five Idahoans receive welfare benefits, almost double the level 10 years ago.” The article goes on to say that nearly 321,700 Idahoans are enrolled in the state’s food stamp, cash assistance, child care, or Medicaid program. But the number has dropped by 10% from the previous year. (Source: magicvalley.com/news/local/govt-and-politics/in-idahoans-receive-welfare-benefits-director-says/article_02ec414e-7cde-1fe3-8a98-001a4bcf887a.html)

a. What percentage of Idahoans receive “welfare benefits”? What percentage received welfare benefits 10 years ago?

b. How many people live in Idaho? How many received “welfare benefits” in the previous year?

c. The article also says that the median wage in Idaho is $11.15 an hour or $23,200 annually. Show how the yearly amount was computed from the hourly rate.

Refer to the bar graph below. Is it possible that more people in Group B than in Group A have the condition under study? If so, provide some sample data to illustrate this. If not, explain why not.
ON YOUR OWN

15 Sometimes the word *odds* is used when referring to risk. However, this word is not used in a consistent way, so it is important to pay close attention to what the person might mean. The mathematical definition of odds is illustrated by this example: If the odds that an event occurs are 3 to 5, then the probability the event occurs is \(rac{3}{3+5} = \frac{3}{8}\). When outcomes are equally likely, the odds of an event are *number of favorable outcomes* to *number of unfavorable outcomes*.

Each statement below uses the word *odds*. For each, decide whether the person probably is using the word according to the mathematical definition or whether they mean something else. Justify your choice.

a. The odds against winning on an American roulette wheel are 37 to 1. On an American roulette wheel, there are 38 spaces and you win if your ball falls into the one you had selected.

b. An article in the *Chronicle of Higher Education* described the odds of getting an interview after submitting an application for a job as university professor as “somewhere in the neighborhood of one in 20 to one in 30.” It then compared this to the odds of surviving the Hunger Games: one in 24. (In the original *Hunger Games*, there are two contestants from each of twelve districts and only one winner.)

(Source: chronicle.com/article/The-Odds-Are-Never-in-Your/144079/)

c. President Obama once said that the odds of completing final treaties in the Middle East “are less than fifty-fifty.”

(Source: www.newyorker.com/reporting/2014/01/27/140127fa_fact_remnick)

d. “Lotteries offer the worst odds in legal gambling—about 55 percent of what people pay for tickets is paid out in prizes. Yet we spend an average of $540 per household on lottery tickets every year …”

(Source: opinionator.blogs.nytimes.com/2014/01/15/playing-the-odds-on-saving/)

e. The National Safety Council gives the “lifetime odds of death” by being bitten or struck by a dog as 1 in 122,216. It gives the odds of dying by any cause as 1 in 1.


16 When absolute risk is low, it is often reported as the number of cases per 1,000 people, per 10,000 people, or per 100,000 people. For example, Problem 4 (page 4) of Investigation 1 says that 54 people out of every 100,000 who get the HPV vaccine have an adverse event.

Read this statement about collisions of autos and deer in Pennsylvania: “The highest rate for 2009 was Fulton County, west of Adams County along the Maryland border, with 16.83 per 10,000 people.” In 2009, Fulton County had 25 deer-related accidents. About how many people lived in Fulton County? (Source: www.dailylocal.com/article/20110310/NEWS/303109986&pager=full_story, March 10, 2011)
17 Write a short summary of what the research summarized below tells you about the placebo effect.

Remifentanil is a very quick acting and effective pain-reliever. However, its effect doesn’t last very long, with half of its usefulness gone in 3 to 4 minutes. In a test of the placebo effect, twenty-two healthy volunteer subjects were hooked up to an IV so that the Remifentanil could be administered. In the first step, before being given Remifentanil, the subjects were subjected to moderate pain (70 on a scale of 0 to 100) caused by a heat source attached to their calf and their reaction measured. In the second step, the subjects were given Remifentanil, but were led to believe that it hadn’t been turned on yet. Their rating of their pain was significantly lower. In the third step, the subjects were told that the drug had been started, and their pain rating dropped about the same amount again as it had in the second step. In the final step, the subjects were told that the drug had been stopped, even though it hadn’t. The subjects reported a considerable increase in pain intensity, almost to as high a level as in the first step when they were receiving no Remifentanil at all. The subject’s brains were scanned during these tests and the activity in the areas of the brain that are involved with pain was consistent with the subjects’ reports of their pain.


18 Look back at your work on Applications Task 3. Does your analysis of these data convince you that loud music, especially played through headphones and MP3 players, is damaging the hearing of teens? Explain.

19 Sometimes you will see risk given in the form “1 in n.” For example, in Problem 2 (page 4) of Investigation 1, the risk of breast cancer for a woman was given as 1 in 8.

a. Refer to the two-way frequency table analyzed in the Think About This Situation (page 2). Write the risk of getting lung cancer for the boys who smoke in the form “1 in n.”

b. Write the risk of getting lung cancer for the boys who do not smoke in the form “1 in n.”

c. Write a procedure that can be used to convert numbers like those in the table to the form “1 in n.”

d. In the Civil War, 77 of 425 Confederate generals were killed in action; 47 of 583 Union generals were killed. (Source: Jim Webb, Born Fighting, Broadway Books, 2004, page 221.) Use your procedure from Part c to convert these risks to the form “1 in n.” Then compare them in a sentence.
Consider the risk of a 16–17-year-old driver being involved in a fatal crash in the next year. (Source: www.cdc.gov/mmwr/preview/mmwrhtml/mm5941a2.htm)

a. In a government report from the U.S. Centers for Disease Control (CDC), the number of 16–17-year-old drivers involved in fatal crashes in a year is given as 16.7 per 100,000. What is the absolute risk, of a 16–17-year-old driver being involved in a fatal crash in the next year?

b. The lowest absolute risk is in New Jersey, 0.000097. Convert this absolute risk to number per 100,000.

c. Write a procedure that can be used to convert absolute risk to number per 100,000.

d. The highest absolute risk is in Wyoming, 0.000596. Use your procedure from Part c to write this risk as number per 100,000.

e. Modify your procedure from Part c and use it to give the risk in Wyoming as number per 10,000.

f. The graph below gives the rates per 100,000 over all states over the years from 1990 to 2008. Write a one-paragraph summary of the most important information shown.
Another statistic used in medical reports is the number needed to treat. This is the number of people who would have to be treated in order to prevent one adverse event. For example, suppose that there are 30 injuries per year for every 1,000 bicyclists who do not wear helmets and 10 injuries per year for every 1,000 who do. The absolute risk reduction is \[
\frac{30}{1,000} - \frac{10}{1,000} = \frac{20}{1,000}.
\]
In other words, 20 injuries are prevented for every 1,000 riders who wear a helmet. To prevent 1 injury, \[
\frac{1,000}{20} = 50
\]
riders would have to wear a helmet. This is the number needed to treat.

a. The number needed to treat is the reciprocal of another statistic. Which one?

b. Millions of Americans take statin drugs to prevent heart attacks and strokes. In one experiment to see how effective atorvastatin was compared to a placebo, 5,168 patients with high blood pressure but normal cholesterol were randomly assigned to get atorvastatin and 5,137 to get a placebo. The numbers who had either coronary heart disease (CHD) or a stroke in three years were 175 for those who took atorvastatin and 258 for those who got a placebo. How many people need to be treated with atorvastatin in order to prevent one case of CHD or stroke in this type of patient? (Source: P.S. Sever et al. “Prevention of Coronary and Stroke Events with Atorvastatin in Hypertensive Patients Who Have Average or Lower-than-Average Cholesterol Concentrations, in the Anglo-Scandinavian Cardiac Outcomes Trial-Lipid-Lowering Arm (ASCOT-LLA): A Multicentre Randomised Trial,” Lancet, Vol. 361, 2003, pp. 1149–1158. www.medicine.ox.ac.uk/bandolier/booth/cardiac/statascot.html)

c. Cryptococcal meningitis is a fungal infection that has a 55% death rate when treated with fluconazole alone. Each year, 957,000 people, mostly in sub-Saharan Africa or Southeast Asia, get this disease. A study found that when amphotericin B is added to the treatment, the death rate drops to 26%. (Source: Nancy Walsh, “Short Ampho Course Works in Cryptococcus, September 27, 2012, www.medpagetoday.com/InfectiousDisease/PublicHealth/34998)

i. How many deaths from cryptococcal meningitis are expected among every 10,000 victims who are treated with fluconazole alone?

ii. How many deaths are expected if all 10,000 receive amphotericin B as well as fluconazole?

iii. What is the number needed to treat?

iv. Use the number needed to treat from part iii in a sentence that explains what it means.
ON YOUR OWN

d. It has been reported that wearing a helmet while skiing/snowboarding reduces the probability of a head injury by 35%. This sounds impressive, but people have pointed out that the risk of a head injury even without a helmet is only about 0.09 per 1,000 outings. Compute and interpret the number needed to treat (wear a helmet) to avoid one head injury.
(Source: www.nhs.uk/news/2011/02February/Pages/head-injury-protection-ski-helmet.aspx)

22 A study was conducted to see how people respond to medical statistics. Each subject was given the following descriptions of three screening tests for cancer.

Test A: If you have this test every 2 years, it will reduce your chance of dying from cancer A by around one third over the next 10 years.

Test B: If you have this test every 2 years, it will reduce your chance of dying from cancer B from around 3 in 1,000 to around 2 in 1,000 over the next 10 years.

Test C: If around 1,000 people have this test every 2 years, 1 person will be saved from dying from cancer C every 10 years.

a. One of these describes the number needed to treat cancer (see Extensions Task 21). Which one?

b. One describes the absolute risk reduction. Which one?

c. One describes the relative risk. Which one?

d. The 306 subjects were asked if they would be likely to accept the test. The percentage who said yes was 80% for Test A, 53% for Test B, and 43% for Test C. Is this rational decision-making? Explain. (Source: Gerd Gigerenzer et al. “Helping Doctors and Patients Make Sense of Health Statistics,” Psychological Science in the Public Interest, Vol. 8, 2008, pp. 53–96.)
ON YOUR OWN

These tasks provide opportunities for you to review previously learned mathematics and to refine your skills in using that mathematics.

23 Being able to work with and understand percentages and ratios is important in many different careers. Consider each of the following situations that require work with percentages or ratios.

a. Some government funding is related to the number of people living in poverty in a particular area. The 2010 population of Caswell County, NC, was 23,719 and 21.7% of the people were living in poverty. [Source: quickfacts.census.gov/qfd/states/37/37033.html] How many people were living in poverty in Caswell County, NC, in 2010?

b. Wisconsin farmers received an average price of $17.60 per 100 pounds for their milk in April 2012, down $1.60 from April 2011. By what percentage did the average price of milk decrease between April 2011 and April 2012? [Source: www.nass.usda.gov/Statistics_by_State/Wisconsin/Publications/Dairy/mkallpri.pdf]

c. The Centers for Disease Control estimates that 1 in 6 people in the United States suffer food poisoning in any given year. At the beginning of 2012, the population of the United States was approximately 312,781,000 people. How many people would you expect to have suffered from food poisoning during 2012? [Sources: www.cdc.gov/foodborneburden/, www.commerce.gov/blog/2011/12/30/census-bureau-projects-us-population-3128-million-new-years-day-2012]

d. Cystic Fibrosis (CF) is a genetic disease that causes mucus to build up and clog some of the organs in the body. The 2012 population of the United States was approximately 313 million people. Approximately 12 million people in the United States carried the defective gene and did not have CF. Suppose you want to say that 1 out of every $x$ people was a carrier of the defective gene for CF without having CF. What is the correct value of $x$? [Source: www.iacfa.org]

24 Determine each number.

a. 36 is 75% of what number?

b. 90 is what percent of 150?

c. What number is $\frac{2}{10}$ of 1% of 4,100?

25 Solve each of the following equations for $n$.

a. $\frac{a}{b} = \frac{1}{n}$

b. $\frac{a}{b} = \frac{n}{1,000}$

c. $\frac{1}{a} = \frac{n + 3}{k}$

d. $4n - 2 = an + 7$

e. $n(5 - k) = 11n + 8$

f. $\frac{8}{an} = a + b$
26 The amount of Tylenol (in mg) that a child should take depends on the child’s weight. Suppose that \( T(w) \) represents the maximum safe dosage for a child weighing \( w \) pounds.

a. \( T(24) = 160 \). Explain what this tells you.

b. The table below gives several weights and the associated dosages. Based on this table of values, would a linear function be a good model for this relationship? Explain.

<table>
<thead>
<tr>
<th>( w )</th>
<th>( T(w) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>160</td>
</tr>
<tr>
<td>48</td>
<td>320</td>
</tr>
<tr>
<td>60</td>
<td>400</td>
</tr>
</tbody>
</table>

c. Based on the table, predict the maximum dosage for a child weighing 80 pounds. Explain your reasoning.

d. In the field of healthcare, functional relationships are conventionally called formulas. In the formula \( T(w) = \frac{20}{3}w \), what does the \( \frac{20}{3} \) tell you?

27 Recall that you can think of an angle as being formed by rotating a ray about its endpoint from an initial position to a terminal position. If the rotation is counterclockwise, the angle has positive measure. If the rotation is clockwise, the angle has negative measure.

The diagram at the right shows an angle with measure \( \theta \) in standard position in a coordinate system. Its initial side \( OA \) coincides with the \( x \)-axis, its vertex is at the origin, and the terminal side of the angle contains the point \( B(x, y) \). Recall that in this context,

\[
\cos \theta = \frac{x}{OB} \quad \text{and} \quad \sin \theta = \frac{y}{OB}.
\]

a. For each point on the terminal side of an angle with measure \( \theta \) in standard position, draw a sketch of the angle. Find \( \cos \theta \) and \( \sin \theta \).

i. \( P(5, 12) \)

ii. \( Q(-6, 4) \)

b. How are these definitions for cosine and sine similar to, and different from, the corresponding right-triangle definitions?

c. Suppose the terminal side of an angle in standard position with measure \( \theta \) is on the axis indicated below. Find \( \cos \theta \) and \( \sin \theta \) in each case.

i. positive \( y \)-axis

ii. negative \( x \)-axis

iii. negative \( y \)-axis

iv. positive \( x \)-axis
ON YOUR OWN

d. Copy the following table. Indicate whether the value of each function is positive or negative in the given quadrant.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>cos θ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sin θ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


e. Describe the pattern of sign change in cos θ as θ increases from 0 to 360°. In sin θ.

28 Recall that if \(10^b = a\), then \(b\) is called the base-10 (or common) logarithm of \(a\). This is denoted \(\log_{10} a = b\) or simply \(\log a = b\).

a. Rewrite each equation using logarithmic notation.
   i. \(10^0 = 1\)
   ii. \(10^2 = 100\)
   iii. \(10^3 = 100,000\)
   iv. \(10^{-2} = 0.01\)

b. Rewrite each equation using exponential notation.
   i. \(\log 10 = 1\)
   ii. \(\log 1,000 = 3\)
   iii. \(\log 0.1 = -1\)
   iv. \(\log 0.001 = -3\)

c. Explain why \(2 < \log 485 < 3\).

d. Complete a copy of the table below by making consecutive integer estimates (see Part c) of the following logarithms. Check your estimates using a graphing calculator or TCMS-Tools.

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log 16)</td>
<td></td>
</tr>
<tr>
<td>(\log 1.6)</td>
<td></td>
</tr>
<tr>
<td>(\log 1,600)</td>
<td></td>
</tr>
<tr>
<td>(\log 3)</td>
<td></td>
</tr>
<tr>
<td>(\log 0.3)</td>
<td></td>
</tr>
<tr>
<td>(\log 30)</td>
<td></td>
</tr>
</tbody>
</table>

e. Use the table in Part d to help determine how the values of \(\log x\) and \(\log 100x\) are related to each other. Justify why this will be true for any value of \(x\).
ON YOUR OWN

29 Without using technology, determine the value of each expression if \( a = -\frac{1}{3} \), \( b = 4 \), and \( c = 3 \). Then check your results using technology.

a. \( \sqrt{a^2} \)

b. \( c^3 - ac \)

c. \( a^{-2} \)

d. \( |b - c| \)

e. \( -6|a| \)

f. \( |b^{-1} + a| \)

30 Gabriel borrows $750 from his parents so that he can start a neighborhood lawn-mowing business. His parents will charge him 6% annual simple interest. Simple interest is calculated using only the actual amount of the loan (or deposit).

a. If Gabriel's parents add the interest to his loan balance on a monthly basis, how much will the interest be each month?

b. Suppose that for the first year, Gabriel does not make any payments to his parents. Make a table showing the loan balance at the end of each month for the first year.

c. Describe how the monthly balance of the loan is changing from one month to the next.

d. Suppose that \( f(n) \) represents the loan balance after \( n \) months and \( f(n + 1) \) represents the loan balance after \( n + 1 \) months, which of the following recursive formulas are correct?

- **Formula I:** \( f(n + 1) = 1.06f(n) \)
- **Formula II:** \( f(n + 1) = 1.005f(n) \)
- **Formula III:** \( f(n + 1) = 0.06f(n) \)
- **Formula IV:** \( f(n + 1) = f(n) + 3.75 \)

e. Write a function rule that represents the amount \( B \) that Gabriel owes his parents as a function of the number of months \( n \) since he borrowed the money.
A Test of Significance

In some cases, it is obvious that there is a difference between two groups. In other cases, it is not so obvious. For example, the U.S. Centers for Disease Control and Prevention (CDC) periodically conducts a survey to track risky behavior by American youth. The two tables below give the responses of 12th-grade students to two questions on the most recent survey. The sample sizes differ somewhat because not every student answers every question. (Source: 2011 Youth Risk Behavior Survey, www.cdc.gov/healthyyouth/yrbs/)

<table>
<thead>
<tr>
<th>Question</th>
<th>12th-Grade Boys</th>
<th>12th-Grade Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rode with a driver who had been drinking alcohol one or more times during the 30 days before the survey?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>488</td>
<td>503</td>
</tr>
<tr>
<td>No</td>
<td>1,292</td>
<td>1,295</td>
</tr>
<tr>
<td>Total</td>
<td>1,780</td>
<td>1,798</td>
</tr>
<tr>
<td>In a physical fight one or more times during the 12 months before the survey?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>616</td>
<td>356</td>
</tr>
<tr>
<td>No</td>
<td>1,191</td>
<td>1,480</td>
</tr>
<tr>
<td>Total</td>
<td>1,807</td>
<td>1,836</td>
</tr>
</tbody>
</table>
THINK ABOUT THIS SITUATION

Think about whether the evidence is comparable that the proportion of all 12th-grade boys who engaged in each risky behavior is different from the proportion of all 12th-grade girls who engaged in the behavior.

a. Compare the proportions of 12th-grade boys and girls who rode with a driver who had been drinking alcohol.

b. Compare the proportions of 12th-grade boys and girls who were in a physical fight.

c. Now suppose that the survey could have asked every 12th-grade boy and girl about these behaviors. For which behavior is there more convincing evidence that the proportion of all 12th-grade boys who engaged in the behavior would be different from the proportion of all 12th-grade girls who engaged in the behavior? Explain your reasoning.

d. If the survey had asked every 12th-grade boy and girl about these behaviors, for which question(s) is it possible that the proportions would be equal? For which question(s) does it seem plausible (believable) that the proportions would be equal?

In this lesson, you will learn how to decide, on the basis of random samples from two different populations, whether it is reasonable to believe that there is a difference in the proportion of all people in each population who have some characteristic. Most of the examples in this lesson will be from the fields of politics and education. You also will reconsider some of the medical situations from the previous lesson.

INVESTIGATION 1

Homogeneous Groups

In this investigation, you will learn how to decide if two groups you are comparing are homogeneous. As you work on the problems in this investigation, look for answers to this question:

What can help you decide whether two random samples were taken from populations with different proportions?

1 Two different groups are called homogeneous if, when they are sorted into the same categories, the proportion of people in the first group who fall into any given category is equal to the proportion of people in the second group who fall into the category. For example, the 2000 and 2010 U.S. Census asked Hispanic residents about their origin. The results are given in the table and graphs on page 38.
a. The top bar graph shows frequency of country of origin on the vertical axis. The bottom bar graph shows percent of country of origin on the vertical axis. Which bar graph best shows how much the total Hispanic population increased in the 10 years between the two censuses? Estimate this increase using the graph.

b. Are the two groups (2000 Hispanics and 2010 Hispanics) homogeneous with respect to origin or are they similar but not homogeneous? Which bar graph shows this best? How would you describe any change?

c. Suppose that the Hispanic population increases from the 2010 Census to the 2020 Census, but the percentage in each category of origin does not change. How would a stacked bar graph with percent on the vertical axis for the 2020 Census differ from the one for the 2010 Census?
You may recall computing an expected number in your previous studies. For example, if you roll a die 42 times, the expected number of times you roll a 4 is 7. That is because the probability of getting a 4 is $\frac{1}{6}$ and $\frac{1}{6} \cdot 42 = 7$. Similarly, the expected number of 4s in 33 rolls is $\frac{1}{6} \cdot 33 = 5.5$. (Expected number does not have to be a whole number.)

2 Suppose you plan to take a random sample of 1,000 Hispanics in the United States. Assume that the distribution of origin has not changed since 2010.

a. What proportion of Hispanics in the United States in 2010 were of Mexican origin? What is the expected number of Hispanics in your sample who will be of Mexican origin?

b. What proportion of Hispanics in the United States in 2010 were of Puerto Rican origin? What is the expected number of Hispanics in your sample who will be of Puerto Rican origin?

c. Now suppose that you take a random sample of 1,000 Hispanics from your state and find 521 Hispanics of Mexican origin. A friend takes a second random sample of 1,500 Hispanics from your state and finds 847 of Mexican origin. What is the best estimate of the proportion of Hispanics in your state who are of Mexican origin?

d. Suppose you plan to take yet another random sample from your state, this time of 500 Hispanics. Based on your work in Part c, what is the best estimate of the expected number of Hispanics of Mexican origin you will find in this sample?

3 The larger group from which a sample is taken is called the population. The proportions in small samples can vary quite a bit from those in the population from which the sample was taken and from each other.

The table at the top of the next page shows two random samples of size 50 taken from the national population of SAT Critical Reading scores.
a. Samples 1 and 2 were taken from the same population. Looking only at these two samples, what is your best estimate of the proportion of all Critical Reading scores that are in the category 500–590?

<table>
<thead>
<tr>
<th>Score</th>
<th>Sample 1</th>
<th>Sample 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>700–800</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>600–690</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>500–590</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>400–490</td>
<td>22</td>
<td>26</td>
</tr>
<tr>
<td>300–390</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>200–290</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td><strong>50</strong></td>
</tr>
</tbody>
</table>

b. What is your best estimate of the expected number of scores in a random sample of size 50 that would fall in the category 300–390?

c. Results from a third random sample are given in the table below.

<table>
<thead>
<tr>
<th>Score</th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>700–800</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>600–690</td>
<td>8</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>500–590</td>
<td>13</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>400–490</td>
<td>22</td>
<td>26</td>
<td>11</td>
</tr>
<tr>
<td>300–390</td>
<td>4</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>200–290</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>50</strong></td>
<td><strong>50</strong></td>
<td><strong>50</strong></td>
</tr>
</tbody>
</table>

i. Match the following three stacked bar graphs (percent on the vertical axis) with the corresponding samples above.

ii. Is it possible that this third sample also is a random sample from the population of all SAT Critical Reading scores? Do you think that is plausible or do you think it probably came from a different population? (Plausible means reasonable to believe.) Explain your reasoning using the data in the table or the stacked bar graphs.
SUMMARIZE THE MATHEMATICS

In this investigation, you learned how to decide whether two groups are homogeneous.

a. What does it mean to say that two groups are homogeneous?

b. How can you tell from a stacked bar graph (percent on the vertical axis) whether two groups are homogeneous? How can you tell from a stacked bar graph (frequency on the vertical axis) whether two groups are homogeneous?

c. How do you compute an expected number?

d. What is the difference between a sample and a population?

Be prepared to share your ideas and computation method with the class.

CHECK YOUR UNDERSTANDING

This table shows the age distribution of people in the United States in 2010.

a. Complete the last column of a copy of this table.

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of People</th>
<th>Proportion of Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
<td>74,181,467</td>
<td>0.2403</td>
</tr>
<tr>
<td>18–44</td>
<td>112,806,642</td>
<td></td>
</tr>
<tr>
<td>45–64</td>
<td>81,489,445</td>
<td>0.2639</td>
</tr>
<tr>
<td>65 and Over</td>
<td>40,267,984</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>308,745,538</strong></td>
<td></td>
</tr>
</tbody>
</table>


b. The following table gives the age distribution for the state of Utah. Are the age distributions for the U.S. and Utah homogeneous? If so, show why. If not, describe the main way that they differ.

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
<td>871,027</td>
</tr>
<tr>
<td>18–44</td>
<td>1,096,191</td>
</tr>
<tr>
<td>45–64</td>
<td>547,205</td>
</tr>
<tr>
<td>65 and Over</td>
<td>249,462</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2,763,885</strong></td>
</tr>
</tbody>
</table>

c. Suppose that you take a random sample of 5,000 people from the U.S. population. Make a table that shows the expected number of people in your sample who come from each of the four age groups.
The Chi-Square Statistic

In the last investigation, you learned that in homogeneous populations, the same proportion of people fall into each category. Frequently, however, you have only a sample from each population. In this investigation, you will learn a method of measuring how different the two samples are. In the next investigation, you will use this statistic to decide whether the samples are so different that you should conclude that the populations from which the samples came are different.

As you complete the following problems, look for an answer to this question:

If you have two samples in which the people are placed into various categories, how can you measure how different the two samples are?

1 Shown below is information on responses from another question on the U.S. Centers for Disease Control and Prevention (CDC) Youth Risk Behavior Survey. This time, only the (approximate) marginal totals are given.

Drank (non-diet) soda daily during the 7 days before the survey?

<table>
<thead>
<tr>
<th></th>
<th>12th-Grade Boys</th>
<th>12th-Grade Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>930</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>2,520</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,725</td>
<td>1,725</td>
<td>3,450</td>
</tr>
</tbody>
</table>

a. Overall, what proportion of the 12th-grade students surveyed drank soda daily? What proportion did not?

b. Fill in the cells of a copy of the table so that the samples of boys and girls are homogeneous. Round your entries to the nearest tenth in this and similar problems.

2 Here is response information from another question on the CDC Youth Risk Behavior Survey. Again, only the marginal totals are given, but this time, the sample sizes are not equal because different numbers of boys and girls had ridden a bike during the previous 12 months.

Wore a bicycle helmet (among students who had ridden a bicycle during the previous 12 months)?

<table>
<thead>
<tr>
<th></th>
<th>12th-Grade Boys</th>
<th>12th-Grade Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rarely or Never</td>
<td></td>
<td>1,825</td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>202</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,160</td>
<td>867</td>
<td>2,027</td>
</tr>
</tbody>
</table>

a. Overall, what proportion of the 12th-grade students surveyed who had ridden a bicycle over the previous 12 months wore a helmet rarely or never? What proportion did wear a helmet?

b. Fill in the cells of a copy of the table so that the samples of boys and girls are homogeneous.
3 The numbers you entered in the cells of the tables in Problems 1 and 2 are called **expected frequencies** or **expected counts**. They show what perfectly homogeneous samples would have looked like.

Reproduced below is the table of **observed frequencies** from Applications Task 4 (page 21) in Lesson 1.

**Observed Frequencies**

<table>
<thead>
<tr>
<th></th>
<th>Women with Melanoma</th>
<th>Women with No Melanoma</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tanning Beds</td>
<td>103</td>
<td>67</td>
<td>170</td>
</tr>
<tr>
<td>Did Not Use Tanning Beds</td>
<td>272</td>
<td>208</td>
<td>480</td>
</tr>
<tr>
<td>Total</td>
<td>375</td>
<td>275</td>
<td>650</td>
</tr>
</tbody>
</table>

a. Overall, what proportion of the women had used tanning beds? What proportion of the women had not used tanning beds?

b. Fill in the cells of a copy of the table below of frequencies expected if the two samples of women were taken from homogeneous populations.

**Expected Frequencies**

<table>
<thead>
<tr>
<th></th>
<th>Women with Melanoma</th>
<th>Women with No Melanoma</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Used Tanning Beds</td>
<td></td>
<td></td>
<td>170</td>
</tr>
<tr>
<td>Did Not Use Tanning Beds</td>
<td></td>
<td></td>
<td>480</td>
</tr>
<tr>
<td>Total</td>
<td>375</td>
<td>275</td>
<td>650</td>
</tr>
</tbody>
</table>

c. Compare the table of observed frequencies with the table of expected frequencies. Do you think the evidence is strong, weak, or nonexistent that women with melanoma are more likely to have used tanning beds than those without melanoma?

4 In 2000 and 2010, the Gallup Poll asked national random samples of 1,000 parents of grades K–12 students,

“Would you say that public education today in grades K through 12 is better, about the same, or worse than when you were a student?”

Here are the results.

**Parent Perceptions of Public Education**

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Better</td>
<td>340</td>
<td>278</td>
</tr>
<tr>
<td>About the Same</td>
<td>124</td>
<td>227</td>
</tr>
<tr>
<td>Worse</td>
<td>536</td>
<td>495</td>
</tr>
<tr>
<td>Total</td>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

**Source:** counts estimated from www.gallup.com/poll/1612/education.aspx
a. Make a table of the frequencies expected if the two samples of parents were taken from homogeneous populations.

b. Compare the table of observed frequencies with the table of expected frequencies. Does the evidence seem strong, weak, or nonexistent that if the 2000 and 2010 Gallup Polls asked all parents, the proportion giving at least one of the responses would have been different?

5 Suppose that you have two samples with marginal totals as given in the table below. For each cell of the table, use the totals and proportional reasoning to write a general formula that can be used to find the expected frequency for homogeneous samples. Compare your formula with that of others. Resolve any differences.

### Expected Frequencies

<table>
<thead>
<tr>
<th></th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Row Total</th>
<th>Column Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Category 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Column Total</strong></td>
<td>Column 1 Total</td>
<td>Column 2 Total</td>
<td>Grand Total</td>
<td></td>
</tr>
</tbody>
</table>

6 Now that you can construct a table of expected frequencies that shows what the two samples would have looked like if they had been perfectly homogeneous, you can measure how different the actual samples are from the perfectly homogeneous ones. For this measurement, you can use the chi-square statistic $\chi^2$. Follow these steps to compute $\chi^2$ in Parts a–c.

**Step 1.** Make a table of expected frequencies.

**Step 2.** For each cell of the table, find the difference $O - E$ between the observed frequency $O$ and the expected frequency $E$.

**Step 3.** Square each difference.

**Step 4.** Divide each squared difference by the expected frequency for that cell.

**Step 5.** Sum up all of the values from Step 4.

a. Compute $\chi^2$ for these data from random samples of 9th- and 12th-graders.

### Favor last-period pep rallies for home football games?

<table>
<thead>
<tr>
<th></th>
<th>9th-Graders</th>
<th>12th-Graders</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>60</td>
<td>58</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>
b. Compute $\chi^2$ for these data from random samples of 9th- and 12th-graders.

Favor an off-campus site for junior-senior prom?

<table>
<thead>
<tr>
<th></th>
<th>9th-Graders</th>
<th>12th-Graders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>39</td>
<td>43</td>
</tr>
<tr>
<td>No</td>
<td>61</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

c. Compute $\chi^2$ for these data from random samples of 9th- and 12th-graders.

Favor double-period math classes?

<table>
<thead>
<tr>
<th></th>
<th>9th-Graders</th>
<th>12th-Graders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>34</td>
<td>48</td>
</tr>
<tr>
<td>No</td>
<td>66</td>
<td>52</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

d. When sample sizes are the same, does $\chi^2$ tend to be larger when the two samples are more different or when the two samples are more alike? Explain your reasoning.

7 Looking back at your work in Problem 6, which of the following gives a formula for the chi-square statistic $\chi^2$? Recall that the symbol $\Sigma$ indicates to add up all of the different values for the expression that follows.

I. $\chi^2 = \sum \left( \frac{O - E}{E} \right)^2$

II. $\chi^2 = \frac{\Sigma (O - E)^2}{E}$

III. $\chi^2 = \sum \frac{(O - E)^2}{E}$

IV. $\chi^2 = \frac{(O - E)^2}{E}$

8 Examine the formula for the chi-square statistic $\chi^2$.

a. If two samples are identical, what is the value of $\chi^2$? Justify your answer.

b. Why is it necessary to square the difference $O - E$? Justify your answer by giving an example of what happens if you do not.
Below are the results of two polls of random samples of high school seniors and juniors, who were asked if they approve of, disapprove of, or do not care about a proposed ban of energy drinks on high school campuses.

### Poll I

<table>
<thead>
<tr>
<th></th>
<th>Seniors</th>
<th>Juniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approve</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Disapprove</td>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td>Do Not Care</td>
<td>60</td>
<td>56</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

### Poll II

<table>
<thead>
<tr>
<th></th>
<th>Seniors</th>
<th>Juniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approve</td>
<td>100</td>
<td>120</td>
</tr>
<tr>
<td>Disapprove</td>
<td>300</td>
<td>320</td>
</tr>
<tr>
<td>Do Not Care</td>
<td>600</td>
<td>560</td>
</tr>
<tr>
<td>Total</td>
<td>1,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

a. Compare the proportions of seniors and juniors who approve, disapprove, and do not care in the two polls.

b. For each poll, use your calculator or computer software to compute a table of expected counts and $\chi^2$.

c. Explain why one value of $\chi^2$ is much larger than the other.
SUMMARIZE THE MATHEMATICS

In this investigation, you learned the meaning of the chi-square statistic $\chi^2$ and how to compute it.

a. How do you compute expected frequencies for a two-way frequency table? What is meant by “expected”? What is the sum of the expected frequencies for each group?

b. What does $\chi^2$ measure? How do you compute it?

c. If two samples are homogeneous—exactly alike in the proportion that fall into each category—what can you say about $\chi^2$?

d. If $\chi^2$ for one pair of samples is larger than that for another pair of samples of the same sample sizes, what can you say about the two pairs of samples?

e. If all else is equal, what effect do larger sample sizes have on the size of the chi-square statistic?

Be prepared to explain your ideas and reasoning to the class.

CHECK YOUR UNDERSTANDING

The Check Your Understanding (page 13) in the previous lesson provided the following information about an outbreak of measles in Colorado. Of 609 children who had been vaccinated, 10 got measles (and those 10 children had been given only one of the two recommended doses). Of the 16 children who were not vaccinated, 7 got measles.

a. Make tables of observed and expected frequencies for this situation. As usual, the explanatory variable defines the columns and the response variable defines the rows.

b. Compute $\chi^2$.

c. Is $\chi^2$ close to 0 or does it seem large? Explain what the value of $\chi^2$ tells you about the two samples.

INVESTIGATION 3

Statistical Significance

You now know that a larger value of $\chi^2$ indicates that the samples are more different from each other than they would be if the value of $\chi^2$ were smaller. But how large does $\chi^2$ have to be before you are convinced that the populations from which the samples were taken are different? After all, you cannot expect random samples, even those taken from the homogeneous populations, to be identical.

As you work on the problems in this investigation, look for answers to this question:

How large does $\chi^2$ have to be before you can conclude that the populations from which the samples were taken are not homogeneous?
The survey in Problem 1 (page 42) of Investigation 2 asked 12th-grade boys and 12th-grade girls whether they drank soda daily. The table of observed frequencies from the actual survey is given below.

Drank soda daily during the 7 days before the survey?

<table>
<thead>
<tr>
<th></th>
<th>12th-Grade Boys</th>
<th>12th-Grade Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>538</td>
<td>392</td>
</tr>
<tr>
<td>No</td>
<td>1,187</td>
<td>1,333</td>
</tr>
<tr>
<td>Total</td>
<td>1,725</td>
<td>1,725</td>
</tr>
</tbody>
</table>

a. Using the table of observed frequencies above and your table of expected frequencies from Problem 1 of the previous investigation, compute $\chi^2$.

b. A value of $\chi^2$ is called statistically significant if it is large enough to reasonably conclude that the populations from which the samples were taken are not homogeneous. Do you think that will turn out to be the case here?

c. One way to determine whether your value of $\chi^2$ is statistically significant is to consider 200 pairs of random samples taken from homogeneous populations, each of which has two categories. Then calculate $\chi^2$ for each of the 200 pairs of samples. Shown below is the distribution for 200 such values of $\chi^2$. What is the largest value of $\chi^2$ that occurred in any of these 200 pairs of samples?

d. Where would the value of $\chi^2$ calculated in Part a be located on this histogram?

e. How does the histogram help you conclude that the value of $\chi^2$ from Part a is large enough to be statistically significant?
f. Select the best conclusion.
   A. If the survey had asked all 12th-grade boys and all 12th-grade girls, it is plausible (reasonable to believe) that the proportions who said they drink soda daily would be equal.
   B. If the survey had asked all 12th-grade boys and all 12th-grade girls, it is not plausible that the proportions who said they drink soda daily would be equal.
   C. Because the survey did not ask all 12th-grade boys and all 12th-grade girls, it is possible that anything could be the case in the two populations, so we cannot come to any reasonable conclusion.

2 Rather than comparing the value of $\chi^2$ calculated from your samples to a histogram each time, statisticians have prepared a table that gives you a quick way to determine whether your $\chi^2$ is statistically significant. Just compare your $\chi^2$ to the appropriate value in the following table, called the critical value. If your $\chi^2$ is larger than the critical value, the difference in the samples is statistically significant. In Problem 7, you will learn more about how the critical values were determined. For now, keep in mind that the larger your value of $\chi^2$, the more evidence you have that the populations from which the two samples were taken are not homogeneous.

### Chi-Square Critical Values

<table>
<thead>
<tr>
<th>Number of Categories</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Value</td>
<td>3.84</td>
<td>5.99</td>
<td>7.81</td>
<td>9.49</td>
<td>11.07</td>
<td>12.59</td>
<td>14.07</td>
</tr>
</tbody>
</table>

The survey reported in Problem 2 (page 42) of the previous investigation asked 12th-graders whether they wore a helmet when riding a bicycle. The table of observed frequencies from the actual survey is given below.

<table>
<thead>
<tr>
<th>Wore a bicycle helmet (among students who had ridden a bicycle during the previous 12 months)?</th>
<th>12th-Grade Boys</th>
<th>12th-Grade Girls</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rarely or Never</td>
<td>1,067</td>
<td>758</td>
<td>1,825</td>
</tr>
<tr>
<td>Yes</td>
<td>93</td>
<td>109</td>
<td>202</td>
</tr>
<tr>
<td>Total</td>
<td>1,160</td>
<td>867</td>
<td>2,027</td>
</tr>
</tbody>
</table>

a. Compute the proportion of each group who rarely or never wore a bicycle helmet. Do you think the difference will turn out to be statistically significant?
b. Using your table of expected frequencies from Problem 2 of the previous investigation and the table above, compute $\chi^2$.
c. How many categories are there for each sample?
d. What critical value should be used for comparison?
e. Is there statistically significant evidence that the samples came from populations that are not homogeneous? Explain.
f. Is the result from Part e consistent with your judgment in Part a?
Refer to the two survey questions examined in the Think About This Situation on page 36.

a. For which survey question do you think the results are most convincing that, if the survey had asked every 12th-grader in the country, the proportions for the two populations (boys and girls) would be different? Which is least convincing?

b. For each survey question, compute $\chi^2$.

c. Is the difference in the two samples statistically significant for either survey question?

d. Is the result from Part c consistent with your judgment in Part a about these polls?

Harris Interactive asked samples of adults (age 18 and over) in various countries, “How important are brand names to you, if at all, when purchasing clothes and fashion accessories?”

The responses are given in the table below, as a percent.

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>China</th>
<th>India</th>
<th>Britain</th>
<th>France</th>
<th>Germany</th>
<th>Spain</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Important</td>
<td>4</td>
<td>19</td>
<td>23</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Important</td>
<td>21</td>
<td>53</td>
<td>51</td>
<td>17</td>
<td>22</td>
<td>19</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>Not That Important</td>
<td>48</td>
<td>24</td>
<td>21</td>
<td>41</td>
<td>49</td>
<td>49</td>
<td>50</td>
<td>51</td>
</tr>
<tr>
<td>Not At All Important</td>
<td>27</td>
<td>4</td>
<td>5</td>
<td>35</td>
<td>26</td>
<td>29</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>Column Total</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Sample Size</td>
<td>2,309</td>
<td>500</td>
<td>500</td>
<td>1,293</td>
<td>1,179</td>
<td>1,058</td>
<td>1,019</td>
<td>1,064</td>
</tr>
</tbody>
</table>

Source: www.harrisinteractive.com

a. Name two countries for which the difference in the percentages between those countries clearly is statistically significant. Explain your reasoning.

b. Name two countries where you think that the difference in the percentages probably will not be statistically significant.

c. For the countries you chose in Part b, convert each percentage to a frequency. Round to the nearest whole number.

d. For the two countries you chose in Part b, compute $\chi^2$.

e. How many categories are there for each sample?

f. What critical value should be used for comparison?

g. Write a conclusion.
5 A chi-square test also can be used to analyze the result of an experiment. In the experiment reported in Problem 2 on page 15, after seven weeks of basic training, 21 of the 200 officer trainees who had received custom orthotics had leg, knee, or foot injury that required he or she stop physical training for two or more days. Sixty-one of the 200 who did not get orthotics had such injuries.

a. Make tables of observed and expected frequencies for this situation.

b. Compute $\chi^2$.

c. Do you have statistically significant evidence that the custom orthotics caused a difference in the proportion of officer trainees who got such injuries? Explain.

6 In a study of conformity, a large group of volunteer Internet users was randomly divided into two groups, a control group and the treatment group. On the Internet, each group was asked the following multiple-choice question (among others):

In which city can you find Hollywood?

- New York
- Las Vegas
- Los Angeles
- San Francisco
- Bombay

Along with the multiple-choice question, the treatment group, but not the control group, was shown the following fabricated information about how others have answered the question.

Others from your community have given the following answers.

<table>
<thead>
<tr>
<th>City</th>
<th>Treatment Group</th>
<th>Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>101</td>
<td>19</td>
</tr>
<tr>
<td>Las Vegas</td>
<td>376</td>
<td>430</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>477</td>
<td>449</td>
</tr>
<tr>
<td>San Francisco</td>
<td>90</td>
<td>1</td>
</tr>
<tr>
<td>Bombay</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The responses to the multiple-choice question are given in the table below.


a. Compute the value of $\chi^2$ for this situation. Is it statistically significant?

b. Write a short conclusion for this study.
In Problem 1, critical values were determined by repeatedly taking pairs of random samples from two populations that are homogeneous. You then observed how large $\chi^2$ tends to be when the populations from which the samples are taken are identical.

For this problem, 200 pairs of random samples were taken from homogeneous populations. In this case, each population had three categories. The chi-square statistic was calculated for each of the 200 pairs of samples. Shown below is the distribution of the 200 values of $\chi^2$.

### a. What is the largest value of $\chi^2$ that occurred in any of these 200 pairs of samples?

### b. What are the major differences between this distribution and the one in Problem 1 Part c?

### c. Use the formula for the chi-square statistic to explain why there are no negative values of $\chi^2$ in this distribution.

### d. Using the table on page 49, find the critical value of $\chi^2$ for the situation where there are three categories.

### e. It is possible to get a value of $\chi^2$ as large as the critical value even when taking two random samples from populations that are identical. But this is not very likely to happen. How many times out of the 200 different pairs of random samples was $\chi^2$ larger than the critical value? What percentage of the time did this happen?

### f. Critical values typically are located so that they cut off the upper 5% of the distribution generated using a method like that above. Thus, values higher than the critical value can happen when samples are taken from homogeneous populations, but this is not likely. The next histogram shows the distribution of the values of $\chi^2$ from 200 different pairs of random samples taken from homogeneous populations. Where would you set the critical value of $\chi^2$ for this distribution? How many categories do you think there were in this case?
SUMMARIZE THE MATHEMATICS

In this investigation, you learned how to decide if a value of $\chi^2$ is statistically significant.

a. How can you tell if the difference in the proportions in two random samples that fall into the various categories is statistically significant?

b. What does it mean when the difference is statistically significant?

c. How are critical values determined?

d. If a difference is not statistically significant, should you conclude that the two populations are homogeneous or should you conclude only that you do not have statistically significant evidence that they are different?

Be prepared to explain your ideas to the class.

CHECK YOUR UNDERSTANDING

A survey collected information from 4,249 young adults age 18 to 28 from 30 countries who had reported crying in the last year. Of the 2,577 women, 1,344 said they felt better after their most recent crying episode, 964 said they felt the same, and 269 said they felt worse. Of the 1,672 men, 851 said they felt better, 674 said they felt the same, and 147 said they felt worse. *(Source: Lauren M. Bylsma, et al. “When is Crying Cathartic? An International Study,” Journal of Social and Clinical Psychology, Vol. 27, 2008, pp. 1165–1187. Numbers estimated from percents.)*

a. Make tables of observed and expected counts so that you can compare the proportion of men and women who fall into the three categories. Round expected counts to the nearest tenth.

b. Compute $\chi^2$.

c. What critical value should be used for comparison?

d. Is the difference in the proportion of men and women who fell into the different categories statistically significant? Write a sentence or two explaining what this means in the given context.
ON YOUR OWN

APPLICATIONS

1 After a round of cut-backs at a manufacturing plant, 18 of 28 younger workers were laid off and 28 of 64 older workers were laid off.

a. Make a table that summarizes the information given.

b. Select the best answer to the question of whether there is evidence of possible age discrimination. Be prepared to describe weaknesses in the other answers.

A. There may be discrimination against older workers because more older workers were laid off than younger workers.

B. There may be discrimination against older workers because a larger proportion of older workers were laid off than younger workers.

C. There may be discrimination against younger workers because more younger workers were laid off than older workers.

D. There may be discrimination against younger workers because a larger proportion of younger workers were laid off.

E. There is no evidence of discrimination because half of the workers were laid off and half were not.

c. Construct a bar graph to convince someone that your answer to Part b is correct. Which type of bar graph did you make? How does the bar graph help the person see that you are correct?
2. The table below shows the ages of the male and female actors who got regular roles in pilot shows for new television series, for those actors whose age could be determined (which was all but 8 males and 5 females).

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of Males</th>
<th>Number of Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>13</td>
<td>7</td>
</tr>
<tr>
<td>10–19</td>
<td>31</td>
<td>29</td>
</tr>
<tr>
<td>20–29</td>
<td>73</td>
<td>95</td>
</tr>
<tr>
<td>30–39</td>
<td>116</td>
<td>88</td>
</tr>
<tr>
<td>40–49</td>
<td>52</td>
<td>21</td>
</tr>
<tr>
<td>50–59</td>
<td>33</td>
<td>19</td>
</tr>
<tr>
<td>60 and over</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>331</strong></td>
<td><strong>269</strong></td>
</tr>
</tbody>
</table>


a. The following two bar graphs show the distribution of ages. Which graph better shows which group (males or females) got more regular roles in pilot shows? Describe this difference between the groups.

b. Are the two groups homogeneous? If so, explain why. If not, explain how they differ.

c. Suppose that over the next three-year period, 1,000 male actors and 800 female actors get regular roles in pilot shows. Assuming that the age distribution remains the same for males and females, what is the expected number of males aged 20–29 who get these jobs? What is the expected number of females in the same age range?
3 This table shows the distribution of scores on the SAT Critical Reading test for 2011 college-bound seniors.

<table>
<thead>
<tr>
<th>Score</th>
<th>Number of Students</th>
<th>Proportion of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>700–800</td>
<td>76,565</td>
<td>0.0465</td>
</tr>
<tr>
<td>600–690</td>
<td>256,676</td>
<td>0.1558</td>
</tr>
<tr>
<td>500–590</td>
<td>480,588</td>
<td>0.2918</td>
</tr>
<tr>
<td>400–490</td>
<td>531,429</td>
<td>0.3267</td>
</tr>
<tr>
<td>300–390</td>
<td>247,836</td>
<td>0.1505</td>
</tr>
<tr>
<td>200–290</td>
<td>54,029</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,647,123</strong></td>
<td></td>
</tr>
</tbody>
</table>


a. What are the missing numbers in the third column of this table?

b. Suppose that you take a random sample of 1,000 scores from this population of SAT Critical Reading scores. What is the expected number of scores in the category 700–800? In the category 400–490?

4 The following table shows the distribution of scores on the SAT Mathematics test for 2011 college-bound seniors.

<table>
<thead>
<tr>
<th>Score</th>
<th>Number of Students</th>
<th>Proportion of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>700–800</td>
<td>111,893</td>
<td>0.0679</td>
</tr>
<tr>
<td>600–690</td>
<td>304,037</td>
<td>0.1846</td>
</tr>
<tr>
<td>500–590</td>
<td>481,170</td>
<td>0.2921</td>
</tr>
<tr>
<td>400–490</td>
<td>498,944</td>
<td>0.3029</td>
</tr>
<tr>
<td>300–390</td>
<td>210,645</td>
<td></td>
</tr>
<tr>
<td>200–290</td>
<td>40,434</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,647,123</strong></td>
<td><strong>1.0000</strong></td>
</tr>
</tbody>
</table>


a. What are the two missing numbers in the third column of this table?

b. Suppose that you take a random sample of 1,000 scores from this population of SAT Mathematics scores. What is the expected number of scores in the category 700–800? In the category 400–490?

c. Make a stacked bar graph (percent on the vertical axis) to compare this distribution with the distribution of SAT Critical Reading scores in Applications Task 3.

d. Are the two populations homogeneous? Explain why or why not.
5 In Problem 4 (page 16) in the previous lesson, you read about a study of young baseball pitchers. It found that 7 of the 103 who had thrown curveballs before the age of 13 were injured seriously enough to have surgery or quit pitching. Of the 187 who had not thrown curveballs before age 13, only 8 were injured seriously enough to have surgery or quit pitching.

a. Make tables of observed and expected frequencies for this situation.

b. Compute \( \chi^2 \).

c. Is \( \chi^2 \) close to 0 or does it seem large? Explain what this means in terms of the two samples.

6 Refer to Applications Task 2 (page 55) about the ages of male and female actors.

a. Add the row totals to a copy of the table. Then, using only the marginal totals, make a table of expected frequencies that shows what perfectly homogeneous groups of males and females would have looked like. Round to the nearest tenth.

b. Compute \( \chi^2 \).

c. Is \( \chi^2 \) close to 0 or does it seem large? Explain what this means in terms of the two groups.

7 Refer to your table of observed frequencies from Applications Task 2 (page 19) in Lesson 1 about the safety of artificial turf.

a. Make a table of expected frequencies for this situation.

b. Compute \( \chi^2 \).

c. How many categories are there for each sample?

d. What critical value should be used for comparison?

e. Is the difference in the two samples statistically significant? Explain how you know.

f. What can you conclude?

8 A worker over age 50 was laid off from a paper products company. He sued his company for age discrimination. Among other evidence presented was this:

In one division of the company, 9 of 22 workers under age 50 were laid off while 19 out of 28 workers age 50 and older were laid off. (Source: Ann E. Watkins, Richard L. Scheaffer, and George Cobb, Statistics: From Data to Decision, Wiley, 2011.)
ON YOUR OWN

a. Make tables of observed and expected frequencies for this situation.
b. Compute $\chi^2$.
c. Is the difference in the proportion of younger and older workers laid off statistically significant? Explain how you know.
d. Given just this evidence, how strong would you judge the worker’s case to be?

9 The Harris Interactive Poll asked about 1,092 adult males and 1,092 adult females in the United States,

“Overall, how satisfied are you with your life nowadays?”

The percentage giving various responses are given in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Male (%)</th>
<th>Female (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Satisfied</td>
<td>25.5</td>
<td>24</td>
</tr>
<tr>
<td>Fairly Satisfied</td>
<td>53.5</td>
<td>59</td>
</tr>
<tr>
<td>Not Very Satisfied</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>Not At All Satisfied</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Source: www.harrisinteractive.com

a. Convert the percentages to frequencies, rounding to the nearest whole number.
b. Compute $\chi^2$.
c. Is there statistically significant evidence that, if Harris had asked all adult males and all adult females in the United States, the distributions of responses would be different? Explain your reasoning.

10 In Investigation 2 Problem 6 (page 11) in Lesson 1, you read about an experiment to determine side effects caused by the use of salmeterol (an ingredient in Advair). Of 1,653 patients who were randomly assigned to receive salmeterol, 35 had been hospitalized. Of 1,622 patients who received the placebo, 16 were hospitalized. The value of $\chi^2$ for this situation is 6.83.

Is there statistically significant evidence that salmeterol caused a change in the rate of hospitalization? Explain.
11 As you read in Extensions Task 21 Part b on page 30, millions of Americans take statin drugs to prevent heart attacks and strokes. In one experiment to see how effective atorvastatin was compared to a placebo, 5,186 patients with high blood pressure but normal cholesterol, were randomly assigned to get atorvastatin and 5,137 to get a placebo. The number of patients who got either coronary heart disease (CHD) or stroke in three years was 175 and 258, respectively. The value of $\chi^2$ for this situation is 17.44.

Is there statistically significant evidence that the use of atorvastatin caused a difference in the proportion of patients with high blood pressure, but normal cholesterol, who got either CHD or stroke in three years?

12 The formula for the distance $d$ between two points, $(x_1, y_1)$ and $(x_2, y_2)$, in the plane is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

a. Use this formula to find the distance between the points (3, 5.2) and (5, 3.2).
b. How is this formula similar to that for the chi-square statistic?
c. How is it different?

13 The chi-square statistic involves a sum of squared differences. You may have learned about other statistics that involve a sum of squared differences. One of these is the standard deviation, which measures how much the values in a set of data vary from their mean. An algorithm to calculate the standard deviation follows.

Step 1. Find the mean $\bar{x}$ of the $n$ values:

$$\bar{x} = \frac{\sum x}{n}$$

Step 2. Find the deviation from the mean for each value:

deviation from the mean = value − mean = $x - \bar{x}$
Step 3. Square each deviation.

Step 4. Find the sum of the squared deviations.

Step 5. Calculate an average of the squared deviations by dividing the sum of the squared deviations by \( n - 1 \). (The reason for using \( n - 1 \) rather than \( n \) is technical.)

Step 6. Take the square root of the average from Step 5.

**a.** Use the above algorithm to compute the standard deviation for each set of grades on chapter tests.

   i. 50, 60, 70, 80, 90
   
   ii. 60, 65, 70, 75, 80

**b.** Use the standard deviation feature of your calculator or computer software to check your answer.

**c.** Which set of grades in Part a has more variability? Does it have the larger standard deviation?

**d.** Look back at your calculations of the standard deviation. Which of the following gives a formula for the standard deviation \( s \)?

\[
 s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}
\]

\[
 s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}
\]

\[
 s = \sqrt{\frac{(\sum x - \bar{x})^2}{n - 1}}
\]

**e.** If all values in a set of data are the same, what is the standard deviation? Use algebraic reasoning to show that this must be the case.

---

14 This table, from Problem 1 (page 38) of Investigation 1, gives the proportion of Hispanics of various origins.

<table>
<thead>
<tr>
<th>Origin</th>
<th>2000 Census</th>
<th>2010 Census</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexican</td>
<td>20,640,711</td>
<td>31,798,258</td>
</tr>
<tr>
<td>Puerto Rican</td>
<td>3,406,178</td>
<td>4,623,716</td>
</tr>
<tr>
<td>Cuban</td>
<td>1,241,685</td>
<td>1,785,547</td>
</tr>
<tr>
<td>Other Hispanic</td>
<td>10,017,244</td>
<td>12,270,073</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>35,305,818</strong></td>
<td><strong>50,477,594</strong></td>
</tr>
</tbody>
</table>

**a.** Use your calculator or computer software to compute a table of expected frequencies that shows how the results would have turned out if the distributions for the two years had been homogeneous. Round to the nearest whole number.

**b.** Use the matrix functions in TCMS-Tools or on your calculator to compute the values of \( O - E \). Are the values of \( O - E \) unusually large or small? Why is that the case?

**c.** Compute \( \chi^2 \). Why is this so large?
Suppose that a polling organization takes a random sample of 100 older U.S. voters and a random sample of 100 younger U.S. voters. The voters are asked whether they approve of the job the President is doing. The results of Poll A are reported in the table at the right.

Another polling organization takes a random sample of 1,000 Republicans and a random sample of 1,000 Democrats and asks the same question. The results of Poll B are reported in the table at the right.

For Poll A, $\chi^2 \approx 0.57$. For Poll B, $\chi^2 \approx 5.70$.

a. Is the result statistically significant in either case? What can you conclude about older and younger voters? About Republicans and Democrats?

b. Are the proportions any different in the two polls? In light of your answer, explain how the result could be statistically significant for one poll and not the other.

c. Compare the values of $\chi^2$. By what factor is the one for Poll B larger than the one for Poll A? Use algebraic reasoning to explain why this must be the case.

### REFLECTIONS

How many times would you have to flip a coin before convincing someone that it was fair? The plot below shows the number of flips of a fair quarter on the horizontal axis and the proportion of times, so far, that the coin came up heads.

a. Was the first flip a head or a tail?

b. Was the 50th flip a head or a tail?
ON YOUR OWN

c. How many heads were in the longest run of consecutive heads?
d. Describe what is happening as the number of flips increases.
e. About how many flips did it take until the proportion of heads stays pretty close to 0.5?
f. Predict what would happen to the pattern in the plot if the person continued flipping the quarter. Explain why this should be the case.

17 Look back at Problem 3 (page 39) of Investigation 1. SAT Critical Reading score is a quantitative variable.
a. How were the data organized to permit SAT Critical Reading scores to be analyzed as a categorical variable using a frequency table?
b. Identify another problem in Investigation 1 for which a similar technique was used.

18 Suppose that the proportions in each category from two samples from Populations A and B differ more than the proportions from two other samples from Populations C and D.
a. Create an example to show that it is not necessarily the case that $\chi^2$ for the samples from Populations A and B will be larger than $\chi^2$ for the samples from Populations C and D.
b. Why is it reasonable that this could be the case?

19 To estimate the proportion of voters in favor of an issue, national polls typically use samples of 1,000 to 1,500 voters. Is that large enough? Some people say that such polls cannot possibly be accurate because no one asked them. Write an explanation to such a person illustrating that the proportion could not change very much whether the poll did ask them or did not.

EXTENSIONS

20 A chi-square statistic also may be used to compare how closely the proportions in a single random sample match specified proportions. This is called a chi-square goodness of fit test. For example, suppose that you have a die and want to test whether it is fair. You roll it 60 times and count the number of times each face lands on top.
a. Complete a copy of this table of expected frequencies for a fair die.

<table>
<thead>
<tr>
<th>Result</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>
b. Here are the observed frequencies of actual results from 60 rolls. Compute $\chi^2$.

<table>
<thead>
<tr>
<th>Result</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>12</td>
<td>9</td>
<td>10</td>
<td>6</td>
<td>11</td>
<td>12</td>
<td>60</td>
</tr>
</tbody>
</table>

c. If $\chi^2$ is larger than the critical value in the Chi-Square Critical Values table reproduced below, the value of chi-square is statistically significant. Is your $\chi^2$ larger than the critical value for 6 categories? Do you have statistically significant evidence that the die is unfair? Explain.

**Chi-Square Critical Values**

<table>
<thead>
<tr>
<th>Number of Categories</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Value</td>
<td>3.84</td>
<td>5.99</td>
<td>7.81</td>
<td>9.49</td>
<td>11.07</td>
<td>12.59</td>
<td>14.07</td>
</tr>
</tbody>
</table>

Indian mythology holds that older snakes get energized from the full moon. This belief prompted researchers to study whether a full moon results in more snakebites of humans.

They looked at 125 consecutive snakebite deaths in Yavatmal, India. Thirty-seven of these deaths occurred during the nine days closest to and including the full moon; 33 occurred five through 10 days after the full moon; and the rest occurred more than 10 days after the full moon. **Source:** Anil K. Batra and Ajay N. Keoliya, “Do Fatal Snakebites Occur More During a Full Moon? An Observational Analysis,” *International Journal of Medical Toxicology & Legal Medicine*, Vol. 7, 2004, www.scribd.com/doc/23986204/Snake-Bites-full-Moon/

a. How many snakebites occurred more than 10 days after the full moon? There are 29 days in the lunar cycle. How many days were in this period?

b. If the lunar cycle has nothing to do with a sample of 125 snakebites, what is the expected number of fatal snakebites in the nine days around and including the full moon? In days five through 10 days after the full moon? On days more than 10 days after the full moon?
c. Using the expected frequencies from Part a, complete a copy of this table.

<table>
<thead>
<tr>
<th>Lunar Cycle</th>
<th>Closest to Full Moon</th>
<th>Five to Ten Days After Full Moon</th>
<th>More than Ten Days After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Number of Snakebites</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected Number of Snakebites</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Compute \( \chi^2 \) as in Extensions Task 20.

e. What is the critical value for this situation? Is the value of chi-square that you computed in Part d statistically significant?

f. What is your conclusion?

22 Learn to spin a penny on a flat surface by holding it on edge with one finger and then flicking it on one side with a finger of the other hand.

a. Spin your penny 50 times. Count the number of times it lands heads and tails.

b. Complete a copy of these tables, filling out the expected frequencies under the assumption that heads and tails are equally likely.

<table>
<thead>
<tr>
<th>Result</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td></td>
</tr>
<tr>
<td>Tails</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Result</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heads</td>
<td></td>
</tr>
<tr>
<td>Tails</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
</tr>
</tbody>
</table>

c. Compute \( \chi^2 \) as in Extensions Task 20.

d. What is the critical value for this situation? Is the value of chi-square that you computed in Part c statistically significant?

e. What is your conclusion? If your result was not statistically significant, should you conclude that heads and tails are equally likely or can you conclude only that you do not have statistically significant evidence that they are different? Explain your reasoning.

f. If possible, combine results with the rest of the class and repeat Parts b through e.

23 When there are only two samples and two categories, another test for statistical significance, the two-proportion \( z \)-test, can be used instead of the chi-square test. For example, in the Check Your Understanding for Investigation 2 on page 47, there are two samples, the vaccinated children and the unvaccinated children. There are two categories, getting measles and not getting measles. Thus, you can use...
either the chi-square test or the two proportion z-test. These two tests are equivalent. To find the two-proportion z-test on a TI-84 calculator, go to the \textit{STAT} menu, arrow over to \textit{TESTS}, and select 6:2-PropZTest... For \( x_1 \), enter the number of people who fall into the first category in the first sample and for \( n_1 \), enter the sample size for the first sample. Continue similarly for the second sample. Select \( p_1 \neq p_2 \), then select \textit{Calculate} and \textit{Enter}. On the resulting screen will be a value of \( z \).

a. Use your calculator to find \( z \) for the data in the Check Your Understanding on page 47. Square this value of \( z \) and compare to the value of \( \chi^2 \) that you computed in the Check Your Understanding.

b. Verify that \( \chi^2 = z^2 \) using the data in Applications Task 5.

c. The following formula for \( z \) looks impressive, but involves only the number of people in the first category in each sample and the sample sizes. (With a lot of algebra, you can prove that \( \chi^2 = z^2 \).)

\[
\begin{align*}
    z &= \frac{x_1 - x_2}{\sqrt{\frac{x_1 + x_2}{n_1 + n_2} \left(1 - \frac{x_1 + x_2}{n_1 + n_2}\right) \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}
\end{align*}
\]

Use this formula to verify the value of \( z \) that you found in Part b.

d. In the formula in Part c, what is given by the expression in the numerator?

REVIEW

24 The circle graph below represents the ethnic breakdown of the population of San Francisco County in 2010. Suppose that someone randomly selected a group of 500 people from San Francisco County in 2010.

San Francisco County Population

<table>
<thead>
<tr>
<th>Percentages 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.9</td>
</tr>
<tr>
<td>33.3</td>
</tr>
<tr>
<td>15.1</td>
</tr>
<tr>
<td>3.6</td>
</tr>
<tr>
<td>6.1</td>
</tr>
</tbody>
</table>

- White Not Hispanic
- Black
- Asian
- Hispanic or Latino Origin
- Other

\textit{Source: factfinder2.census.gov/faces/tableservices/jsf/pages/productview.xhtml}

a. What is the expected number of Black people in the group?
b. What is the expected number of Hispanics/Latinos in the group?
c. What is the expected number of Asians in the group?
ON YOUR OWN

d. Suppose another person selected a group of people that was \( n \) times the size of the original sample. How would your answers to Parts a–c change? Support your answer with mathematical reasoning.

25 Darius has a savings account that earns 2.35% annual interest compounded annually. His current account balance is $5,278. Assume that Darius does not withdraw or deposit any money and that he leaves all earned interest in the account.

a. How much interest will Darius earn during the first year? What will his account balance be at the end of the first year?

b. During the second year, will Darius earn more interest or the same amount of interest than he did during the first year? Explain your reasoning.

c. What will his account balance be five years from now?

d. Explain how the balance in Darius’s account changes from one year to the next.

e. Explain why the function rule \( B(t) = 5,278(1.0235^t) \) can be used to determine the balance in Darius’s account after \( t \) years.

26 In the diagram at the right \( \angle AOB \) is in standard position; its initial side coincides with the positive \( x \)-axis, its vertex is the origin and its terminal side contains \( \overrightarrow{OB} \). As you may recall, angles in standard position in a coordinate system can be measured in terms of revolutions of the initial side of the angle, in degrees, and in radians. Recall that a \textbf{radian} is the measure of a \textbf{central angle} that intercepts an arc equal in length to the radius of the circle. In the diagram at the right, the radian measure of \( \angle AOB \) is 1.

a. If the initial side of an angle is rotated one complete revolution, then the measure of the angle is \( 2\pi \) radians. Explain why this is the case.

b. Complete each of the following statements.

i. \( \text{revolution} = \text{90°} = \text{???? radians} \)

ii. \( \text{革命} = \text{?? degrees} = \frac{\pi}{3} \text{ radians} \)

c. Complete a copy of the following table to show equivalent revolution, degree, and radian measurements.

<table>
<thead>
<tr>
<th>Revolution/Degree/Radian Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolutions</td>
</tr>
<tr>
<td>Degrees</td>
</tr>
<tr>
<td>Radians</td>
</tr>
</tbody>
</table>
27 The histogram below displays the ages of the Green Bay Packers roster on July 5, 2011. On that day, the roster had 85 players.

![Histogram](image)

Source: [www.packers.com/team/players.html](http://www.packers.com/team/players.html)

a. Three players were 20 or 21 years old. How can this be determined by looking at the histogram?

b. What percentage of the players were 28 or 29 years old?

c. What percentage of the players were at least 30 years old?

d. What percentage of the players were younger than 36 years of age?

e. When asked the median age of the players on the team, Dario said that it was 29 years. Do you agree or disagree with Dario? Explain your reasoning.

28 Consider the circle with radius 5 that is centered at the origin.

![Circle](image)

a. If \( \angle AOB = 60^\circ \), find the coordinates of point B.

b. Describe the location of another point \( B_1 \) on the circle that has the same \( x \)-coordinate as point B. What is the relationship between the \( y \)-coordinates of the two points?

c. Describe the location of another point \( B_2 \) on the circle that has the same \( y \)-coordinate as point B. What is the relationship between the \( x \)-coordinates of the two points?
ON YOUR OWN

29 Shown at the right is a circle of radius 1, called a unit circle. \( \angle POQ \) is an angle in standard position with radian measure \( \theta \).
   a. Why is \( \cos \theta \) the x-coordinate of point \( Q \)?
   b. Why is \( \sin \theta \) the y-coordinate of point \( Q \)?
   c. Suppose the terminal side of \( \theta \), \( \overrightarrow{OQ} \), is in the quadrant given below. Determine, in terms of \( \theta \), the x- and y-coordinates of point \( Q \).
      i. Quadrant II
      ii. Quadrant III
      iii. Quadrant IV
   d. Suppose the radius of a circle is equal to the unit segment on a tape measure.

   Imagine wrapping the tape measure counterclockwise around the circle, starting where \( \overrightarrow{OP} \) meets the circle and noting the point on the tape measure where \( \overrightarrow{OQ} \) meets the circle. The arc length measured by the tape measure will be the radian measure of the angle. Wrapping the tape measure around the circle (in both directions from point \( P \)) produces a correspondence between points on the tape measure and points on the circle. Consider the coordinates of the points on the circle under the above correspondence. How does this method construct cosine and sine functions with domain all real numbers?

30 The amount of force \( f \) (in pounds) needed to break a board (with fixed width and thickness) is a function of the length \( \ell \) (in inches) of the board. For one type of board, the function rule is \( f(\ell) = \frac{20}{\ell} \).
   a. As the length of the board increases, how does the amount of force needed to break the board change? How is your answer reflected in a graph of \( f(\ell) \)?
   b. Evaluate \( f(12) \). What does your answer tell you about this situation?
   c. Solve \( f(\ell) = 0.5 \). What does this solution tell you about this situation?
   d. The length of board that Peter is trying to break is twice that of the one that his older brother just broke. How does the force needed by Peter compare to the force needed by his brother?
Mrs. Takemura surveyed the 440 juniors and seniors at Sayre High School and asked them whether they worked for pay in the previous week. The results are reported in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Worked for Pay</th>
<th>Did Not Work for Pay</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Juniors</td>
<td>100</td>
<td>140</td>
<td>240</td>
</tr>
<tr>
<td>Seniors</td>
<td>90</td>
<td>110</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td>190</td>
<td>250</td>
<td>440</td>
</tr>
</tbody>
</table>

Suppose that you randomly pick one of these students to interview.

a. Compute this probability: \( P(\text{student worked for pay the previous week}) \)

b. Compute this probability: \( P(\text{student was a junior}) \)

c. Is a randomly selected senior or a randomly selected junior more likely to have worked for pay the previous week?

d. The notation for the probability that the student worked for pay the previous week, if the student you picked turned out to be a senior is \( P(\text{worked for pay the previous week | was a senior}) = \frac{90}{200} \). The vertical line | is read “given that.” Write the notation for the probability that the student worked for pay the previous week, if the student you picked turned out to be a junior.

e. Suppose that you pick a student who worked for pay the previous week. Compute the probability that the student was a junior, \( P(\text{was a junior | worked for pay the previous week}) \).

f. Compute \( P(\text{was a senior | worked for pay the previous week}) \).

g. Is a randomly selected student who worked for pay the previous week more likely to be a junior or a senior? Explain.
Texting has increasingly become the way in which people, young and older, communicate with one another. About 77% of 12th-graders text, with about half of those sending at least 60 text messages per day. (Source: pewinternet.org/Reports/2010/Teens-and-Mobile-Phones/Summary-of-findings.aspx)

Suppose that you take a random sample of 12th-grade students who text and categorize them according to two variables,

- whether they send at least 60 texts per day and
- whether they text their parents every day.

The results are given in the table below.

<table>
<thead>
<tr>
<th>Send At Least 60 Text Messages Daily</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>275</td>
<td>65</td>
<td>340</td>
</tr>
<tr>
<td>No</td>
<td>35</td>
<td>225</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>310</td>
<td>290</td>
<td>600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Text Parents Daily</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>275</td>
<td>65</td>
<td>340</td>
</tr>
<tr>
<td>No</td>
<td>35</td>
<td>225</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>310</td>
<td>290</td>
<td>600</td>
</tr>
</tbody>
</table>
THINK ABOUT THIS SITUATION

Think about whether there is an association between the number of texts sent by 12th-graders and whether they text their parents daily.

a. What proportion of 12th-graders who send at least 60 texts daily also text their parents daily? Is the proportion the same for 12th-graders who do not send at least 60 texts daily?

b. What proportion of 12th-graders who text their parents daily also send at least 60 texts daily? Is the proportion the same for 12th-graders who do not text their parents daily?

c. Are the two variables *send at least 60 texts daily* and *text parents daily* associated in this sample? Or are they independent variables; that is, does knowing whether a teen does one of these change the probability that he or she does the other? Explain your answer.

d. Does it seem plausible, just from looking at the sample, that the two variables *send at least 60 texts daily* and *text parents daily* would be independent in the population of all 12th-grade students who text?

e. Does it seem plausible, from knowing what you know about texting among 12th-graders, that the two variables *send at least 60 texts daily* and *text parents daily* would be independent in the population of all 12th-grade students? Explain.

In the previous two lessons, you analyzed situations where there were two groups, such as two samples taken at random from two different populations. In this lesson, you will examine single samples, classified on two different categorical variables. The previous table is an example of such a situation.

INVESTIGATION 1

Diagnostic Testing

If someone is suspected of having a medical problem, often he or she first is given an inexpensive and quick diagnostic test, sometimes called a *screening test*. These tests are not always very accurate, so if a positive result is obtained, a more accurate but usually more expensive, time-consuming, or invasive diagnostic test is given.

As you work on the problems in this investigation, look for answers to this question:

*What are the statistical characteristics of a good diagnostic test?*
1 When a pregnant woman receives a blow to the abdomen, doctors must check for pelvic free fluid (FF), which is associated with internal injury. A fast and inexpensive way to do this is with an ultrasound. But sometimes an ultrasound gives the wrong result. A more expensive and invasive test is available, which never gives the wrong result. To check the accuracy of the ultrasound test, 328 pregnant women who had received a blow to the abdomen were given both tests. The results are given in the table below. *(Source: E. L. Ormsby, et al., “Pelvic Free Fluid: Clinical Importance for Reproductive Age Women with Blunt Abdominal Trauma,” *Ultrasound in Obstetrics & Gynecology*, September 2005, pp. 271–278.)*

<table>
<thead>
<tr>
<th>Result of Ultrasound</th>
<th>Positive for FF</th>
<th>Negative for FF</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF Actually Present</td>
<td>14</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>FF Actually Absent</td>
<td>15</td>
<td>290</td>
<td>305</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>299</td>
<td>328</td>
</tr>
</tbody>
</table>

a. A **false positive** occurs when a test shows a positive result but is wrong. How many false positives were there? Explain the meaning of a false positive for the woman tested.

b. A **false negative** occurs when a test shows a negative result but is wrong. How many false negatives were there? Explain the meaning of a false negative for the woman tested.

c. Looking only at those who had FF, what proportion got a positive ultrasound? This is called the **sensitivity** of the test.

d. Looking only at those who did not have FF, what proportion got a negative ultrasound? This is called the **specificity** of the test.

e. Looking only at those who got a positive ultrasound, what proportion actually had FF? This is called the **positive predictive value (PPV)**.

f. Looking only at those who got a negative ultrasound, what proportion actually did not have FF? This is called the **negative predictive value (NPV)**.

g. What do you see as the major statistical strengths and weaknesses of the ultrasound test for FF? Share your ideas with your classmates. Resolve any differences.
2 The statistics defined in Problem 1 Parts c–f may be thought of as conditional probabilities. That is, they are all of the form:

\[
\frac{\text{cell count}}{\text{row (or column) total}}
\]

Conditional probability uses special notation. For example, the sensitivity of a test is the probability of a positive test result when the condition actually is present. This is written \( P(\text{positive test} \mid \text{condition present}) \). Here, \( P \) stands for probability and the vertical line \( \mid \) is read, “given that” or “if it is true that.”

Here is the complete definition of the sensitivity of a test:

\[
\text{sensitivity} = P(\text{positive test} \mid \text{condition present}) = \frac{\text{count with condition and positive test}}{\text{total with condition}}
\]

a. Write a definition of the specificity of a test using conditional probability.

b. Write a definition of the positive predictive value (PPV) using conditional probability.

c. Write a definition of the negative predictive value (NPV) using conditional probability.

d. If a diagnostic test is a good one, which of the four conditional probabilities defined above should be close to 0? Which should be close to 1?

3 Upon admission to college, students often must take placement tests in English and mathematics. Students who “fail,” for example, the mathematics placement test must retake high school level mathematics (called remedial mathematics) before being able to take the mathematics required to graduate from college. Sometimes students fail the mathematics placement test because they did not bother to review basic ideas of algebra and geometry before taking the test. These students are placed into remedial mathematics classes even though they are capable of doing college level mathematics. The hypothetical table below shows the results for a random sample of 100 entering freshmen who did not bother to review for the placement test.

<table>
<thead>
<tr>
<th></th>
<th>Fail Placement Test (Test Positive for Remedial Math)</th>
<th>Pass Placement Test (Test Negative for Remedial Math)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Need Remedial Math</td>
<td>25</td>
<td>11</td>
<td>36</td>
</tr>
<tr>
<td>(Have Condition)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do Not Need Remedial Math</td>
<td>26</td>
<td>38</td>
<td>64</td>
</tr>
<tr>
<td>Total</td>
<td>51</td>
<td>49</td>
<td>100</td>
</tr>
</tbody>
</table>

Use this table in completing the following questions and tasks.
a. Define a *positive test* as failing the placement test (testing positive for needing remedial mathematics) and define *condition present* as needing remedial mathematics.

i. How many false positives were there? What is the meaning of a false positive for a student?

ii. How many false negatives were there? What is the meaning of a false negative for a student?

b. Compute the sensitivity and specificity of this placement test for a student who does not review for it. Report each value in a sentence that explains its meaning.

c. Suppose a student fails the test. What is the probability that he or she actually needs to take remedial mathematics? What is the name for this value?

---

4 When the prevalence of a disease in the population is very low, such as HIV infection or certain cancers, there is controversy about the benefits of screening everyone for the disease. In this problem, you will see why this is the case.

The table below shows what is likely to happen if all roughly 300,000,000 Americans each were given an inexpensive enzyme immunoassay screening test for HIV infection.

<table>
<thead>
<tr>
<th></th>
<th>Test Positive</th>
<th>Test Negative</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have HIV</td>
<td>1,339,370</td>
<td>4,030</td>
<td>1,343,400</td>
</tr>
<tr>
<td>Do Not Have HIV</td>
<td>4,479,849</td>
<td>294,176,751</td>
<td>298,656,600</td>
</tr>
<tr>
<td>Total</td>
<td>5,819,219</td>
<td>294,180,781</td>
<td>300,000,000</td>
</tr>
</tbody>
</table>


a. How many false positives were there? Explain the consequence of a false positive for the person tested.

b. How many false negatives were there? Explain the consequence of a false negative for the person tested.

c. What is the sensitivity of this test? Explain the meaning of this statistic, in the context of this test.

d. What is the specificity of this test? Explain the meaning of this statistic, in the context of this test.

e. What is the positive predictive value (PPV)? Use this value in a sentence explaining to a person what his or her positive test might indicate.

f. What is the negative predictive value (NPV)? Use this value in a sentence explaining to a person what his or her negative test might indicate.

g. Based on your results, explain why people are reluctant to recommend universal screening for HIV.
SUMMARIZE THE MATHEMATICS

In this investigation, you learned how to evaluate a diagnostic test statistically.

a. What is a false positive? What is a false negative?

b. Write the sensitivity and specificity of a test as conditional probabilities. Describe what each of them tells you.

c. If you test positive for a disease, why is the positive predictive value (PPV) the single best statistic for you to know? Write the PPV as a conditional probability.

d. A screening test for prostate cancer, called a PSA, is available, but the proportion of positive test results that are false is around 75%.
   i. Which of the four statistics defined in Problem 1 can you compute from this information?
   ii. If the PSA test is positive, a biopsy must be performed, which often has harmful side effects. Why do many people believe that the test should not be given to all men?

Be prepared to share your responses and explain your ideas.

CHECK YOUR UNDERSTANDING

High intracranial pressure (inside the skull) typically is a result of an injury to the head and can be very dangerous. A screening test for high intracranial pressure was proposed many years ago, based on the data in the following observations. This simple and non-invasive test involves observing the retinal vein to see if it is pulsating. Pulsation is normal and so would be considered a negative test result.

<table>
<thead>
<tr>
<th></th>
<th>Pulsation Absent (Positive Test)</th>
<th>Pulsation Present (Negative Test)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>High Intracranial Pressure (Condition Present)</strong></td>
<td>43</td>
<td>0</td>
<td>43</td>
</tr>
<tr>
<td><strong>Normal Intracranial Pressure (Condition Absent)</strong></td>
<td>18</td>
<td>128</td>
<td>146</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>61</td>
<td>128</td>
<td>189</td>
</tr>
</tbody>
</table>

a. How many false positives were there among these 189 people? Explain the meaning of a false positive for the person tested.

b. How many false negatives were there? Explain the meaning of a false negative for the person tested.

c. What is the sensitivity of this test?

d. What is the specificity of this test?

e. What is the positive predictive value (PPV)? Use this value in a sentence explaining to a person the meaning of his or her positive test.

f. What is the negative predictive value (NPV)? Use this value in a sentence explaining to a person the meaning of his or her negative test.

g. All in all, does this seem to be a good screening test? Explain your reasoning.

INVESTIGATION 2

Independence

In this investigation, you will review how to determine whether two events are independent and extend this idea to the independence of two categorical variables. As you work on the problems in this investigation, look for answers to these questions:

What does it mean for two categorical variables to be independent?

How can you assess if two categorical variables are independent?

A graduate student in marine science observed bottlenose dolphins in Sarasota Bay, Florida that engaged in “depredation.” That means that the dolphins were stealing or damaging bait or prey already captured by human fishing gear. She classified each of these dolphins (that she could identify) by the categorical variables of age and sex, as shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>13</td>
<td>22</td>
</tr>
</tbody>
</table>

a. Suppose that you select one of the 22 dolphins at random. What is the probability that the randomly selected dolphin is young? That is, find \( P(\text{young}) \).

b. What is the probability that the randomly selected dolphin is young, if you are told that it is female? Use the notation \( P(\text{young} \mid \text{female}) \) in your answer.

c. What is the probability that the randomly selected dolphin is young, given that it is male? Write your answer using notation like in Part b.

d. Compare your probabilities in Parts b and c. What can you conclude?

e. Compare \( P(\text{female} \mid \text{young}) \) and \( P(\text{male} \mid \text{young}) \). What can you conclude?

2 In some situations, knowing which category a person falls into on one variable helps you better predict which category they fall into on a second variable. For example, suppose you want to estimate the probability that a teen has long hair. If you are told that the teen is female, then that fact helps you—the probability is higher than if you had been told the teen is male. The variables gender and hair length are associated (or, dependent). On the other hand, suppose that you want to estimate the probability that a teen is carrying a cell phone. Then knowing whether the teen is male or female probably does not help you at all—males and females are equally likely to be carrying cell phones. The variables of gender and cell phone possession are independent. [Source: www.pewinternet.org/Reports/2010/Social-Media-and-Young-Adults/ Part-2/1-Cell-phones.aspx]

a. In the situation in Problem 1, does knowing whether one of these dolphins is young or adult help you predict whether it is male? That is, are the variables independent or associated? Explain your answer.

b. If the table had been like the one below, would the variables be independent or associated? Explain your answer.

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>

3 Complete a copy of the following tables using students in your class. First, agree on how to classify hair length and fingernail length into either long or short.

<table>
<thead>
<tr>
<th>Long Fingernails</th>
<th>Short Fingernails</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Hair</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short Hair</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Likes Broccoli</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does Not Like Broccoli</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
a. Are the two variables in the first table independent or associated? In the second table?

b. Using the marginal totals for your class, fill in the cells of a copy of the first table to make the variables hair length and fingernail length independent.

c. Using the marginal totals for your class, fill in the cells of a copy of the second table to make the variables independent.

4 In Problem 3, you may have checked for independence using this rule:
Two events $A$ and $B$ are independent if $P(A) = P(A | B)$ (assuming $P(B) \neq 0$).

a. Suppose that you randomly select one of the dolphins from those in the table of Problem 1. Using the rule above with male as event $A$ and adult as event $B$, determine whether the events male and adult are independent. Choose the correct word to interpret this result: Adult dolphins are [more, less, equally] likely to be male than are dolphins overall.

b. Use the above rule to determine whether the events female and young are independent events. Interpret your result.

An equivalent way to check whether events $A$ and $B$ are independent is to verify that $P(A \text{ and } B) = P(A) \cdot P(B)$. Again, suppose that you randomly select one of the dolphins from those in the table in Problem 1 reproduced below.

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>13</td>
<td>22</td>
</tr>
</tbody>
</table>

c. Using this rule with male as event $A$ and adult as event $B$, determine whether the events male and adult are independent events.

d. Use this rule to determine whether the events female and young are independent events.

5 Two categorical variables are called independent if $P(A \text{ and } B) = P(A) \cdot P(B)$ for all categories $A$ that make up the first variable and all categories $B$ that make up the second variable.

a. Use this rule to determine whether the categorical variables of sex and age are independent in the table in Problem 1 reproduced above.

b. Use this rule to determine whether the categorical variables of sex and age are independent in the sample given in the following table from Problem 2.

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Female</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>
c. When two categorical variables are independent, the columns are proportional. That is, each column is a multiple of the first column. (The multiplier does not have to be a whole number.) Is that the case for the table in Part b? Is that the case for the table in Problem 1?

d. When two categorical variables are independent, it also is true that each row is a multiple of the first row. Is that the case for the table in Part b? Is that the case for the table in Problem 1?

e. Make a bar graph for these data, using male and female as the two bars. How can you tell from this graph alone that the two variables of sex and age are independent?

6 The numbers given in the table below are the marginal totals.

<table>
<thead>
<tr>
<th></th>
<th>Wearing Jeans</th>
<th>Not Wearing Jeans</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teen</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adult</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>20</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Fill in the four cells of a copy of the table so that the variables are independent.
b. Fill in the four cells of a copy of the table so that the variables are not independent.
c. Is there more than one possible answer for Part a? For Part b?

7 Write a formula that uses only the marginal totals and can be used to fill in the cells of a table to make the variables independent. Check your formula using this table.

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>30</td>
<td>70</td>
<td>100</td>
</tr>
</tbody>
</table>

8 The table below shows only the marginal totals for the dolphin study in Problem 1. Use your formula from Problem 7 to complete a copy of the table with values that would make the two variables of age and sex independent. (The values will not be whole numbers.)

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adult</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>13</td>
<td>22</td>
</tr>
</tbody>
</table>
SUMMARIZE THE MATHEMATICS

In this investigation, you reviewed independent events and learned how to tell if two categorical variables are independent.

a. Describe two methods of deciding whether two events are independent.

b. How can you tell if two categorical variables are independent in a population? Give at least two ways.

c. If you have only the marginal totals, how can you determine frequencies for each cell that make the variables independent?

d. If two categorical variables are independent, does knowing which category a person belongs to on the first variable help you predict which category he or she belongs to on the second variable? Explain your reasoning.

Be prepared to explain your ideas to the class.

CHECK YOUR UNDERSTANDING

In a study of 203 young male/female couples, each partner privately was asked whether they had ever cheated and whether they suspected or knew that their partner had cheated. The results are given in the tables below.

<table>
<thead>
<tr>
<th></th>
<th>Female Cheated</th>
<th>Female Had Not Cheated</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male Right</td>
<td>29</td>
<td>162</td>
<td>191</td>
</tr>
<tr>
<td>Male Wrong</td>
<td>9</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>38</td>
<td>165</td>
<td>203</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Male Cheated</th>
<th>Male Had Not Cheated</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female Right</td>
<td>24</td>
<td>138</td>
<td>162</td>
</tr>
<tr>
<td>Female Wrong</td>
<td>35</td>
<td>6</td>
<td>41</td>
</tr>
<tr>
<td>Total</td>
<td>59</td>
<td>144</td>
<td>203</td>
</tr>
</tbody>
</table>


a. If you select a couple at random, what is the probability that the male cheated? The female cheated?

b. Are males or females more likely to be wrong about whether their partner cheated?

c. Do males or females do a better job of detecting cheating? Justify your answer using conditional probability notation.

d. Are the events male cheated and female right independent? Show the computations needed to justify your answer.
e. Are the two variables independent or associated for the first table? For the second table?

f. In context, describe the association in the first table.

g. Fill in the cells of a copy of the following table so that the two variables are independent. Do the actual frequencies and those under the assumption of independence appear somewhat similar or quite different?

<table>
<thead>
<tr>
<th></th>
<th>Male Cheated</th>
<th>Male Had Not Cheated</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female Right</td>
<td></td>
<td></td>
<td>162</td>
</tr>
<tr>
<td>Female Wrong</td>
<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>Total</td>
<td>59</td>
<td>144</td>
<td>203</td>
</tr>
</tbody>
</table>

INVESTIGATION 3

The Chi-Square Test of Independence

As you probably found in Problem 3 (page 78) of the last investigation, in your small sample of males and females, the variables likes/does not like broccoli and male/female are not independent. However, the mathematical definition that you used there may seem too strict. Perhaps if you had a larger group, the proportions who liked broccoli would be more equal.

As you work on the problems in this investigation, look for answers to this question:

How can you tell whether it is plausible that two categorical variables are independent in the population from which a random sample was taken?

Reproduced below is the text messaging table from the lesson Think About This Situation (page 70), which shows a random sample of 12th-grade students who text, categorized according to two variables, whether they send at least 60 texts per day and whether they text their parents everyday.

<table>
<thead>
<tr>
<th>Send At Least 60 Text Messages Daily</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>275</td>
<td>65</td>
<td>340</td>
</tr>
<tr>
<td>No</td>
<td>35</td>
<td>225</td>
<td>260</td>
</tr>
<tr>
<td>Total</td>
<td>310</td>
<td>290</td>
<td>600</td>
</tr>
</tbody>
</table>
a. Suppose that the variables send at least 60 texts daily and text parents daily
had been independent in the sample. Fill in a copy of the following table,
showing what the cell frequencies would have been. Round to the nearest
tenth. As in Lesson 2, these values, which do not have to be whole numbers,
are called expected frequencies.

<table>
<thead>
<tr>
<th>Send At Least 60 Text Messages Daily</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Text Parents Daily</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>340</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>260</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>310</td>
<td>290</td>
<td>600</td>
</tr>
</tbody>
</table>

b. Compare the expected frequencies and the observed frequencies. Do the
differences seem relatively large or small?

c. Compute the chi-square statistic $\chi^2$ using the method in Lesson 2.

d. Reproduced below is the table on page 49 in Lesson 2. For the 2 by 2 tables
(2 rows and 2 columns) in this lesson, you use the critical value for two
categories. To what critical value should your value of $\chi^2$ from Part c be
compared?

<table>
<thead>
<tr>
<th>Chi-Square Critical Values</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Categories</td>
<td>3.84</td>
<td>5.99</td>
<td>7.81</td>
<td>9.49</td>
<td>11.07</td>
<td>12.59</td>
<td>14.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Categories</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Value</td>
<td>3.84</td>
<td>5.99</td>
<td>7.81</td>
<td>9.49</td>
<td>11.07</td>
<td>12.59</td>
<td>14.07</td>
</tr>
</tbody>
</table>

e. Is the value of $\chi^2$ statistically significant?

f. Pick what you believe to be the best interpretation. Be prepared to defend
your choice.

A. Conclude that the two variables send at least 60 texts daily and text parents daily are
independent in the population of all 12th-grade students.

B. Conclude that it is plausible that the two variables are independent in the population.

C. Conclude that the variables are associated because it is not plausible that the two variables are independent in the population.

D. Conclude that the variables are associated because it is not possible that the two variables are independent in the population.
2 Baseball fans can be fanatic about statistics. In fact, baseball fans are interested in such things as whether there is any association between whether the game was played at night or in the day and whether the home team or visiting team won. The table below gives the outcomes from a random sample of 1,000 games.

<table>
<thead>
<tr>
<th></th>
<th>Day Game</th>
<th>Night Game</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Team Wins</td>
<td>242</td>
<td>316</td>
<td>558</td>
</tr>
<tr>
<td>Visiting Team Wins</td>
<td>183</td>
<td>259</td>
<td>442</td>
</tr>
<tr>
<td>Total</td>
<td>425</td>
<td>575</td>
<td>1,000</td>
</tr>
</tbody>
</table>

a. Using the marginal totals in the table above, complete a table of expected frequencies, giving the cell frequencies for the case that the variables day/night game and home/visiting team wins were independent in the sample. Round to the nearest whole number.

b. Compare the expected frequencies and the observed frequencies. Do the differences seem relatively large or small?

c. Compute the chi-square statistic $\chi^2$.

d. To what critical value in the Chi-Square Critical Values table should your value of $\chi^2$ be compared?

e. Is the value of $\chi^2$ statistically significant?

f. Pick the best interpretation. Be prepared to defend your choice.
   A. Conclude that the two variables are independent in the population.
   B. Conclude that it is plausible that the two variables are independent in the population.
   C. Conclude that the variables are associated because it is not plausible that the two variables are independent in the population.
   D. Conclude that the variables are associated because it is not possible that the two variables are independent in the population.

3 Refer to Problem 3 (page 77) from Investigation 2, where you collected data from your class. Assume that you can think of your class as a random sample of all students your age. For each of the two tables, complete the following.

a. Compute the chi-square statistic $\chi^2$.

b. Is the value of $\chi^2$ statistically significant?

c. Write a conclusion in context describing the relationship between the variables.
SUMMARIZE THE MATHEMATICS

In this investigation, you learned how to conduct and interpret a chi-square test of independence.

a. When is it appropriate to use the chi-square test of independence?

b. Describe how to conduct this test, including how to make a table of expected frequencies.

c. When results are not statistically significant, why do you conclude that it is plausible that the two variables are independent rather than concluding that the variables are independent?

d. When results are statistically significant, what can you conclude about the two variables?

Be prepared to share your ideas and reasoning with the class.

CHECK YOUR UNDERSTANDING

The following table is based on a random sample of people age 18 and older who said they were planning a vacation this summer. They were asked if they would bring a laptop computer with them.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bringing a Laptop</td>
<td>354</td>
<td>307</td>
<td>661</td>
</tr>
<tr>
<td>Not Bringing a Laptop</td>
<td>356</td>
<td>320</td>
<td>676</td>
</tr>
<tr>
<td>Total</td>
<td>710</td>
<td>627</td>
<td>1,337</td>
</tr>
</tbody>
</table>

Source: frequencies estimated from The Harris Poll, Americans Work on Their Vacation, 2011

a. Compute a table of expected frequencies.

b. Compare the expected frequencies and the observed frequencies. Do the differences seem relatively large or small?

c. Compute the chi-square statistic $\chi^2$.

d. Is the value of $\chi^2$ statistically significant? Describe what that means for this situation.
### ON YOUR OWN

1. Unwanted, unexpected, and often fraudulent email, called *spam*, typically is sent to try to get your money. Your email program most likely has a junk email filter, which places spam in a special folder or sends it directly to the trash. The filter decides which messages are spam based on the occurrence of certain words that are frequently used in spam. For example, the default settings on one widely used spam filter sends all messages that contain the phrase “hey bro,” to the spam folder. *(Source: spamassassin.apache.org/tests_3_3_x.html)*

Of course, sometimes “hey bro,” or any other indicator of spam occurs in real email (called *ham*). Here are the results for a random sample of 100 messages, which were sorted using one spam filter.

<table>
<thead>
<tr>
<th></th>
<th>Sent to Spam Folder</th>
<th>Sent to Inbox</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actually Spam</td>
<td>36</td>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>Actually Ham</td>
<td>2</td>
<td>53</td>
<td>55</td>
</tr>
<tr>
<td>Total</td>
<td>38</td>
<td>62</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Define a false positive as sending ham to the spam folder. How many false positives were there? Explain what a false positive means for the recipient of the message.

b. How many false negatives were there? Explain what a false negative means for the recipient.

c. What is the sensitivity of the spam filter? Explain the meaning of this statistic in this context.

d. What is the specificity of the spam filter? Explain the meaning of this statistic in this context.

e. What is the positive predictive value (PPV)? Use this value in a sentence explaining its meaning in this context.

f. What is the negative predictive value (NPV)? Use this value in a sentence explaining its meaning in this context.

g. All in all, does this seem to be a good spam filter?

---

### APPLICATIONS

- **Program:** MMH Core Plus Math
- **Component:** TCMS-SE
- **Vendor:** Six Red Marbles
- **Grade:** 12

---

**Lesson 3 | The Relationship Between Two Variables**
ON YOUR OWN

2 Alzheimer’s disease (AD) causes progressive cognitive impairment. A diagnostic blood test for AD recently was proposed. An evaluation of this blood test involved 50 known AD patients and 40 controls who did not have AD. Of those who had AD, 48 tested positive on the blood test and 2 tested negative. Of those without AD, 3 tested positive on the blood test and 37 tested negative. (Source: Eric Nagele et al. “Diagnosis of Alzheimer’s Disease Based on Disease-Specific Autoantibody Profiles in Human Sera,” PLOS ONE, Vol. 6, 2011.)

a. Make a table that summarizes the information above.

b. How many people who did not have AD tested positive on the blood test? What is the name for this kind of test result?

c. How many people who did have AD tested negative on the blood test? What is the name for this kind of test result?

d. With a serious disease, it is important that a high proportion of those who have the disease be detected by the test. Was that the case here? What is the name for the proportion of cases that are detected by a test?

e. It also is important that a high proportion of those who do not have the disease should test negative. Was that the case here? What is the name for this proportion?

f. What is the positive predictive value (PPV)? Use this value in a sentence explaining to a person what his or her positive test suggests.

g. What is the negative predictive value (NPV)? Use this value in a sentence explaining to a person what his or her negative test suggests.

h. All in all, do you think this is a good test? Explain.

3 About 5.4 million people in the United States, almost all age 65 and over, have Alzheimer’s disease (AD). In the Check Your Understanding on page 41, you saw that 40,267,984 people in the United States are age 65 and over. In Parts a–d, you will complete a copy of the following table, which reflects what would happen if all people age 65 and over were tested and the probabilities estimated from the data in Applications Task 2 are correct. (Source: www.alz.org/alzheimers_disease_facts_and_figures.asp)

<table>
<thead>
<tr>
<th></th>
<th>Tested Positive</th>
<th>Tested Negative</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has AD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does Not Have AD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Tables are not provided in the text, but should be inserted here.)
ON YOUR OWN

a. About how many people in the United States age 65 and over do not have AD? Fill in the row marginal totals and the grand total in your table.

b. What percentage of the people who do have AD would you expect to test positive, based on the results in Applications Task 2? Fill in the number of the people who do have AD that you would expect to test positive. Then fill in the number that you would expect to test negative.

c. Repeat Part b for the people who do not have AD.

d. Finally, fill in the column totals.

e. What would be the negative consequences of universal screening for AD using this test?

4 In a classic psychology experiment, preschool children individually were put in a room with a marshmallow (or other treat they selected). They were told that the investigator had to leave for 15 minutes and if they did not eat the marshmallow until he returned, they would be given a second marshmallow and then they could eat both. Suppose that this experiment is replicated with 4 year-olds to see if there is any relationship between ability to wait and whether the child can tie their shoes. The results are given below.

<table>
<thead>
<tr>
<th></th>
<th>Waited</th>
<th>Ate Treat</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can Tie Shoes</td>
<td>8</td>
<td>20</td>
<td>28</td>
</tr>
<tr>
<td>Cannot Tie Shoes</td>
<td>14</td>
<td>35</td>
<td>49</td>
</tr>
<tr>
<td>Total</td>
<td>22</td>
<td>55</td>
<td>77</td>
</tr>
</tbody>
</table>

a. If you select one of these children at random, what is the probability that they can tie their shoes?

b. If you select one of the children at random from those who can tie their shoes, what is the probability that they were able to wait until the investigator returned?

c. If you select one of the children at random from those who waited, what is the probability that they can tie their shoes?

d. Are the events can tie shoes and waited independent? Explain how you know.

e. Are the categories can/cannot tie shoes and waited/ate treat independent? Explain how you know.
Two experts were asked to decide (separately) whether sculptures and drawings supposedly by Henry Moore were genuine or questionable. All had been offered on eBay over a 21-month period. The results are given in the table below. Suppose that these results are typical.

![Evaluator 1 and Evaluator 2 table]

Source: Joseph Gastwirth and Wesley Johnson, "Dare You Buy a Henry Moore on eBay?" Significance, March 2011, pp. 10–14.

a. If you submit a randomly selected Moore sculpture or drawing from eBay to these two evaluators, what is the probability that they both think it is genuine?

b. Which evaluator would you say is more suspicious about the authenticity of the sculptures and drawings? Justify your answer.

c. What is the probability that Evaluator 1 thinks a Moore sculpture or drawing is genuine, given that Evaluator 2 thinks it is genuine?

d. Are the events Evaluator 1 thinks a Moore sculpture or drawing is genuine and Evaluator 2 thinks it is genuine independent? Show the computations needed to justify your answer.

e. Are the variables Evaluator 1 and Evaluator 2 independent? Justify your answer.

f. Should the variables Evaluator 1 and Evaluator 2 be independent or associated in this situation? Explain your reasoning.

g. Fill in the cells of a copy of the following table so that the variables Evaluator 1 and Evaluator 2 are independent.
Researchers wanted to establish whether irritable bowel syndrome (IBS) and generalized anxiety disorder (GAD) are associated maladies. They collected the following information from a random sample of 2,005 people contacted by phone.

<table>
<thead>
<tr>
<th></th>
<th>GAD</th>
<th>No GAD</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBS</td>
<td>18</td>
<td>91</td>
<td>109</td>
</tr>
<tr>
<td>No IBS</td>
<td>63</td>
<td>1,833</td>
<td>1,896</td>
</tr>
<tr>
<td>Total</td>
<td>81</td>
<td>1,924</td>
<td>2,005</td>
</tr>
</tbody>
</table>


a. Complete a table of expected frequencies, rounding to the nearest tenth, for a chi-square test of independence.

b. Compute the chi-square statistic $\chi^2$.

c. To what critical value should your value of $\chi^2$ from Part b be compared?

d. The article concludes that IBS and GAD are strongly associated. Do you agree? Explain why or why not.

e. Write a conclusion in context describing the relationship between the variables.

North Carolina keeps track of traffic accidents. The table below shows all 150 crashes in 2010 involving both a teen driver and a death. The accidents are sorted on two variables, whether alcohol was involved and whether the accident was in town or near town.

<table>
<thead>
<tr>
<th></th>
<th>In Town</th>
<th>Near Town</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Alcohol Involved</td>
<td>30</td>
<td>91</td>
<td>121</td>
</tr>
<tr>
<td>Alcohol Involved</td>
<td>10</td>
<td>19</td>
<td>29</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>110</td>
<td>150</td>
</tr>
</tbody>
</table>

Source: buffy.hsrc.unc.edu/crash/datatool.cfm

a. Complete a table of expected frequencies, rounding to the nearest tenth, for a chi-square test of independence.

b. Compute the chi-square statistic $\chi^2$.

c. To what critical value should your value of $\chi^2$ from Part b be compared?

d. Is the association statistically significant? Explain.

e. Write a conclusion in context describing the relationship between the variables.
This chart shows the 36 equally likely outcomes when rolling a pair of six-sided fair dice. The entry (1, 2), for example, represents the outcome *first die landed with a 1 on top and second die landed with a 2 on top.*

\[
\begin{array}{cccccc}
(1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\
(2, 1) & (2, 2) & (2, 3) & (2, 4) & (2, 5) & (2, 6) \\
(3, 1) & (3, 2) & (3, 3) & (3, 4) & (3, 5) & (3, 6) \\
(4, 1) & (4, 2) & (4, 3) & (4, 4) & (4, 5) & (4, 6) \\
(5, 1) & (5, 2) & (5, 3) & (5, 4) & (5, 5) & (5, 6) \\
(6, 1) & (6, 2) & (6, 3) & (6, 4) & (6, 5) & (6, 6)
\end{array}
\]

Suppose that you roll this pair of dice.

a. Find \( P(\text{sum is 6}). \)

b. Find \( P(\text{sum is 6} \mid \text{doubles}). \)

c. Find \( P(\text{doubles} \mid \text{sum is 6}). \)

d. Find \( P(\text{sum is 6 and doubles}). \)

e. Find \( P(\text{sum is 6 or doubles}). \)

In Investigation 2, you used this definition of independent events: Events \( A \) and \( B \) are independent if \( P(A \text{ and } B) = P(A) \cdot P(B). \)

a. Refer back to your work in Connections Task 8. Are the events *doubles* and *sum is 6* independent according to this definition? Show your work.

b. Are the events *doubles* and *get a 1 on the first die* independent according to this definition? Show your work.

c. You may have learned the following equivalent way of showing that two events are independent: Events \( A \) and \( B \) are independent if \( P(A) = P(A \mid B) \) (assuming \( P(B) \neq 0 \)). Describe in words the meaning of this definition.
d. Using the definition in Part c, determine if the events *doubles* and *sum is 6* are independent. Use *doubles* as event *A*.

e. Using the definition in Part c, determine if the events *doubles* and *get a 1 on the first die* are independent. Use *doubles* as event *A*.

f. There is a third equivalent way of showing that two events are independent: Events *A* and *B* are independent if \( P(B) = P(B \mid A) \) (assuming \( P(A) \neq 0 \)). Describe in words the meaning of this definition.

g. Using the definition in Part f, determine if the events *doubles* and *sum is 6* are independent. Use *doubles* as event *A*.

h. Using the definition in Part f, determine if the events *doubles* and *get a 1 on the first die* are independent. Use *doubles* as event *A*.

10 Use the definition of independent events in Connections Task 9 Part c to determine whether the following events are independent. Interpret your results.

a. The events *teen driver* and *speed related* in the table below, which shows all crashes with fatalities in North Carolina in 2010

<table>
<thead>
<tr>
<th></th>
<th>Not Speed Related</th>
<th>Speed Related</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Teen Driver</td>
<td>793</td>
<td>281</td>
<td>1,074</td>
</tr>
<tr>
<td>Teen Driver</td>
<td>90</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>Total</td>
<td>883</td>
<td>341</td>
<td>1,224</td>
</tr>
</tbody>
</table>

Source: buffy.hsrc.unc.edu/crash/datatool.cfm

b. The events *junior* and *going to the homecoming dance* in the table below

<table>
<thead>
<tr>
<th></th>
<th>Going to the Homecoming Dance</th>
<th>Not Going to the Homecoming Dance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Junior</td>
<td>250</td>
<td>150</td>
<td>400</td>
</tr>
<tr>
<td>Senior</td>
<td>125</td>
<td>75</td>
<td>200</td>
</tr>
<tr>
<td>Total</td>
<td>375</td>
<td>225</td>
<td>600</td>
</tr>
</tbody>
</table>

11 Use the following table of hypothetical data to think about the difference between a chi-square test of homogeneity and a chi-square test of independence.

<table>
<thead>
<tr>
<th></th>
<th>Teen Driver</th>
<th>Driver Not a Teen</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speeding Involved</td>
<td>42</td>
<td>46</td>
<td>88</td>
</tr>
<tr>
<td>Speeding Not Involved</td>
<td>20</td>
<td>34</td>
<td>54</td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
<td>80</td>
<td>142</td>
</tr>
</tbody>
</table>
ON YOUR OWN

a. Suppose that these data were collected by taking a random sample of 62 accidents from accidents involving teen drivers and a random sample of 80 accidents from accidents not involving a teen driver. Do you use a chi-square test of homogeneity or a chi-square test of independence? Why?

b. Now suppose that these data were collected by taking a single random sample of 142 accidents and sorting them according to the two categorical variables. Do you use a chi square test of homogeneity or a chi-square test of independence? Why?

c. A friend says that \( \frac{62}{142} \), or about 43.7\%, is a reasonable estimate of the percentage of all accidents that involve a teen driver. Is that correct if the data were collected as described in Part a? Is that correct if the data were collected as described in Part b?

REFLECTIONS

12 Often a screening test is judged positive or negative based on a cut-off value. For example, a fasting blood glucose test for diabetes is considered positive if it is at or above the cut-off value of 126 mg/dL. Where to place the cut-off value is a matter of experience and good judgment.

Suppose that a higher measurement on a test is more evidence of the condition than a lower measurement. A lab suggests lowering the cut-off value. What is likely to be the effect of lowering the cut-off value on the sensitivity? On the specificity? On the PPV? On the NPV?

13 In a good screening test, should the two variables of condition present/absent and test positive/negative be independent or associated? Explain your reasoning.

14 Describe the similarities and the differences between a chi-square test of homogeneity and a chi-square test of independence.

EXTENSIONS

15 In your work in Investigation 2 and in Connections Task 9, you saw three equivalent ways to verify that events A and B are independent.

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

\[
P(A) = P(A | B)
\]

\[
P(B) = P(B | A)
\]
Each way is based on the definition of conditional probability:

\[ P(A | B) = \frac{P(A \text{ and } B)}{P(B)} \]

a. Refer to Connections Task 8. Use the definition of conditional probability to find \( P(\text{doubles} | \text{sum is 4}) \). Then use it to find \( P(\text{sum is 4} | \text{doubles}) \).

b. Use the definition of conditional probability to show that if it is true that \( P(A) = P(A | B) \), then it also is true that \( P(A \text{ and } B) = P(A) \cdot P(B) \).

c. Use the definition of conditional probability to show that if it is true that \( P(A \text{ and } B) = P(A) \cdot P(B) \), then it is also true that \( P(A) = P(A | B) \).

16 Sometimes one or both of the categorical variables has more than two categories. In other words, the table is larger than 2 rows and 2 columns. For example, the following table gives information about a random sample of traffic crashes in Virginia in 2011.

<table>
<thead>
<tr>
<th>Alcohol Related</th>
<th>Not Alcohol Related</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed Limit Exceeded</td>
<td>26</td>
<td>103</td>
</tr>
<tr>
<td>Safe Speed Exceeded</td>
<td>6</td>
<td>109</td>
</tr>
<tr>
<td>No Speed Violation</td>
<td>48</td>
<td>858</td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>1,070</td>
</tr>
</tbody>
</table>


a. Compute a table of expected frequencies, in the usual way, for a chi-square test of independence.

b. Compute the value of \( \chi^2 \) in the usual way.

c. The larger the table, the larger the critical value will be. To find the critical value in such cases, you compute the degrees of freedom (df): Multiply the number \( R \) of rows minus 1 times the number \( C \) of columns minus 1.

\[ df = (R - 1)(C - 1) \]

Then look up the critical value in the following table. Note that the critical values are the same as in the table in Lesson 2. However, this table uses Degrees of Freedom rather than Number of Categories.

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Value</td>
<td>3.84</td>
<td>5.99</td>
<td>7.81</td>
<td>9.49</td>
<td>11.07</td>
<td>12.59</td>
<td>14.07</td>
</tr>
</tbody>
</table>

Is the value of \( \chi^2 \) from Part b statistically significant?

d. Where does the largest difference between the observed and expected frequency occur? What can you conclude from this test?
ON YOUR OWN

17 A random sample of college students were asked to identify their favorite season of the year and whether they preferred to spend most of their time outside, inside, or did not care. The table below is based on their responses.

<table>
<thead>
<tr>
<th></th>
<th>Outside</th>
<th>Inside</th>
<th>Did Not Care</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring</td>
<td>127</td>
<td>162</td>
<td>152</td>
<td>441</td>
</tr>
<tr>
<td>Summer</td>
<td>95</td>
<td>143</td>
<td>129</td>
<td>367</td>
</tr>
<tr>
<td>Fall</td>
<td>49</td>
<td>75</td>
<td>83</td>
<td>207</td>
</tr>
<tr>
<td>Winter</td>
<td>35</td>
<td>62</td>
<td>54</td>
<td>151</td>
</tr>
<tr>
<td>Total</td>
<td>306</td>
<td>442</td>
<td>418</td>
<td>1,166</td>
</tr>
</tbody>
</table>

a. Compute a table of expected frequencies for a chi-square test of independence.

b. Compute the value of $\chi^2$.

c. Use the rule in Extensions Task 16 to compute the degrees of freedom? Use the table in that task to find the critical value.

d. Is the value of $\chi^2$ from Part b statistically significant?

e. Where does the largest difference between the observed and expected frequency occur? Write a conclusion for this situation.

18 In this unit, when you used technology to determine the value of $\chi^2$, you may have noticed that you were also given the degrees of freedom $df$ and a $P$-value $p$. The $P$-value is the probability of getting a value of $\chi^2$ as large or larger than the one you computed for your sample if it had been taken from a population where the two variables are independent. If the $P$-value is smaller than 0.05, the value of $\chi^2$ is statistically significant.

a. Conduct a chi-square test for the data in Extensions Task 16 using technology. What are the degrees of freedom?

b. What is the $P$-value? Is the $P$-value smaller than 0.05?

c. Is the value of $\chi^2$ statistically significant?
19 Refer to Extensions Task 18. Conduct a chi-square test for the data in Extensions Task 17 using technology.

a. What are the degrees of freedom?

b. What is the $P$-value? Is the $P$-value larger than 0.05?

c. Is the value of $\chi^2$ statistically significant?

20 Angles of rotation can be measured in degrees, radians, and revolutions. Fill in each blank with the appropriate value. Recall that 1 revolution = $360^\circ = 2\pi$ radians.

a. $270^\circ = \ldots \text{ radians} = \ldots \text{ revolution}

b. $\ldots ^\circ = \frac{5\pi}{4} \text{ radians} = \ldots \text{ revolution}

c. $\ldots ^\circ = \ldots \text{ radians} = \frac{5}{6} \text{ revolution}

d. $\ldots ^\circ = 3\pi \text{ radians} = \ldots \text{ revolutions}

21 Chet has recently accepted a new job as a salesperson. He will earn a base annual salary of $36,000 plus 3% commission on his sales during the year.

a. Explain why the function rule $P(s) = 36,000 + 0.03s$, where $s$ is Chet’s total annual sales, can be used to determine Chet’s annual earnings.

b. Determine the value of $P(130,000)$ and explain what it tells you.

c. For what value of $s$ will $P(s) = 43,200$? What does this tell you about Chet’s earnings?

d. Write a function rule that could be used to determine Chet’s monthly pay (salary and commission) for any amount of sales during a month.

22 Recall that a coordinate system is divided into four quadrants that are numbered in a counterclockwise direction as shown in the diagram at the right. If the terminal side of an angle in standard position is determined by a counterclockwise rotation of its initial side, the angle has positive measure. If the rotation is clockwise, the angle has negative measure.
a. Consider the following measures of angles in standard position. Based on the location of the terminal side of each angle, place each angle measure in the correct column of a copy of the table below.

<table>
<thead>
<tr>
<th>Angle Measure</th>
<th>Quadrant I</th>
<th>Quadrant II</th>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{8}$ radians</td>
<td>$\frac{10\pi}{6}$ radians</td>
<td>$\frac{2\pi}{3}$ radians</td>
<td>$\frac{6\pi}{5}$ radians</td>
<td></td>
</tr>
</tbody>
</table>

b. In Quadrant I, which of the angles have terminal sides in the same position?

c. In Quadrant II, which of the angles have terminal sides in the same position?

d. In Quadrant III, which of the angles have terminal sides in the same position?

e. In Quadrant IV, which of the angles have terminal sides in the same position?

23 Without using technology, evaluate each of the following. Then check your answer using technology.

a. $\log_{10} 10,000$

b. $3 \log 100$

c. $\log 10^8$

d. $\log 0.01$

e. $10^{\log 50}$

24 In one class, students were asked to solve the equation $x(x + 7) = x(2x + 3)$. Victoria and Taylor came up with two different solutions to this equation. Their work is shown below. Who is correct? What mistake did the other student make?

**Victoria**

\[
x(x + 7) = x(2x + 3) \\
x + 7 = 2x + 3 \\
4 = x
\]

**Taylor**

\[
x(x + 7) = x(2x + 3) \\
x^2 + 7x = 2x^2 + 3x \\
O = x^2 - 4x \\
O = x(x - 4) \\
x = 0 \text{ or } x = 4
\]
The table and graph below provide information about invoices for new roofs installed by Piedmont Roofing last month.

<table>
<thead>
<tr>
<th>Roof Area (in sq. ft)</th>
<th>Invoice Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>$3,773</td>
</tr>
<tr>
<td>750</td>
<td>$5,457</td>
</tr>
<tr>
<td>1,000</td>
<td>$5,825</td>
</tr>
<tr>
<td>896</td>
<td>$3,906</td>
</tr>
<tr>
<td>1,500</td>
<td>$6,923</td>
</tr>
<tr>
<td>1,000</td>
<td>$3,380</td>
</tr>
<tr>
<td>1,295</td>
<td>$6,083</td>
</tr>
<tr>
<td>729</td>
<td>$2,849</td>
</tr>
<tr>
<td>1,720</td>
<td>$9,663</td>
</tr>
<tr>
<td>1,175</td>
<td>$8,014</td>
</tr>
</tbody>
</table>

a. Without using technology, estimate what it would cost someone to have Piedmont Roofing install a new roof with an area of 1,225 square feet. Be prepared to explain how you determined your estimate.

b. Using technology such as TCMS-Tools data analysis, determine the equation of the least squares regression line. Then explain what the slope and y-intercept of the least squares regression line tell you about the cost of having Piedmont Roofing install a new roof.

Miguel is riding on a Ferris wheel that has a 10-m radius and turns in a counterclockwise direction. Miguel is currently in the “3 o’clock” position. Consider Miguel’s position in relation to the vertical line through the center of the wheel.

a. Determine his directed distance from the vertical line after he has rotated 90° counterclockwise.

b. Determine his directed distance from the vertical line after he has rotated 225° counterclockwise.

c. For what rotations (between 0° and 360°) will Miguel be 5 meters to the left or right of the vertical line through the center of the wheel?
ON YOUR OWN

d. Which graph at the right shows Miguel’s directed distance from the vertical line through the center of the wheel as he travels through one revolution?

e. Which of the following function rules matches the graph you chose in Part d?

Rule I \[ f(x) = 10 \cos x \]
Rule II \[ f(x) = \cos 10x \]
Rule III \[ f(x) = \cos x + 10 \]

27 At the beginning of an experiment to test a bacteria-killing substance, 8,000 bacteria were present. The day after the substance was introduced, only 6,000 bacteria were present.

a. What percent of the bacteria were still present after one day? By what percentage did the number of bacteria decrease?

b. If the same percent change continues each day, how many bacteria will be present at the end of the second day?

c. If NOW represents the number of bacteria present on any day and NEXT represents the number of bacteria present on the following day, write a rule involving NOW and NEXT that gives the relationship between the two quantities.

d. Assume that the number of bacteria present is an exponential function of the number of days \( d \) since the substance was introduced. Which function rule best models this situation? Be prepared to explain your reasoning.

I. \[ f(d) = 8,000(0.25)^d \]

II. \[ g(d) = 8,000(0.75)^d \]

III. \[ h(d) = 8,000 + 0.75^d \]

IV. \[ j(d) = 8,000 + 0.25^d \]

e. When will the total number of bacteria present be 1,000? Explain how you determined your answer.
In this unit, you learned the meaning of terms often used in the media to compare the proportion of people in different groups who have some characteristic. These terms include absolute risk, absolute risk reduction, relative risk, and statistically significant. You also learned why it is important not to rely on anecdotal evidence, but to look for evidence from well-designed experiments. You used the chi-square statistic to decide whether the difference in the proportions that fall into each category in two random samples is statistically significant. Finally, you used a chi-square test to decide whether it is plausible that two categorical variables are independent in the population from which the random sample was taken.

The tasks in this final lesson will help you review and solidify your understanding of key ideas and methods for making sense of categorical data.

1. About 62% of Americans have a pet, with women more likely than men to have one. A Harris Poll of 601 male adult pet owners and 753 female pet owners asked if they considered their pet to be a member of the family. Eighty-five percent of the males said yes, 12% said no, and 3% were not sure. Of the female pet owners, 95% said yes, 3% said no, and 2% were unsure. You may consider these independent random samples of male and female pet owners. (Source: “Pets Really Are Members of the Family,” The Harris Poll, June 10, 2011)

a. Summarize the information in a two-way table of observed frequencies. Let the columns be men and women pet owners.

b. Make a bar graph that best compares the proportion of men and women pet owners who gave the different responses. Does the difference in the responses appear to be significant or do the two groups appear almost homogeneous?

c. To show what homogeneous samples of male and female pet owners would have looked like, make a table of expected frequencies. Round to the nearest tenth.

d. Compute $\chi^2$ to summarize the difference between male and female responses.
e. How many categories are there for each sample? What critical value should be used for comparison?

f. Is the difference in the proportions of male and female pet owners who gave the different responses statistically significant? Explain how you know, and what it means in this context.

2 Tonsils, at the back of the throat, are organs of the lymphatic system. They may help the immune system fight disease. Sometimes, when they are chronically inflamed, doctors remove them, with a surgical procedure called a tonsillectomy. Sweden keeps a register of surgery done on people under the age of 20. During a 15-year period, 27,284 young people had a tonsillectomy. For each of them, five young people the same age, sex, and county of residence were randomly selected to serve as controls. The control group ended up with 136,401 young people. After about 23 years, 47 of the tonsillectomy group and 169 of the control group had had a premature heart attack. (Source: Imre Janszky, et al. “Childhood Appendectomy, Tonsillectomy, and Risk for Premature Acute Myocardial Infarction—A Nationwide Population-Based Cohort Study,” European Heart Journal, online access June 1, 2011.)

a. What is the explanatory variable in this study? What is the response variable? Was this study an experiment? Explain why or why not.

b. Make a table of observed frequencies.

c. What is the absolute risk for each group? Why is the incidence of heart attack so low in both groups?

d. Compute the absolute risk reduction of a heart attack. Use this in a sentence for parents.

e. Compute the relative risk of a heart attack. Use this in a sentence for parents.

f. Assume that you can consider the two groups equivalent to randomly selected samples from all young people with and without tonsillectomies. Compute χ².

g. Is the difference in the proportions of people in the two groups who have heart attacks statistically significant? Explain how you know.

h. Write a short article for your school newspaper about this study. Use and explain the terms absolute risk, relative risk, and statistical significance.

3 Post-traumatic stress disorder (PTSD) is a highly anxious state that can develop after exposure to psychological trauma. Researchers at the U.S. Department of Veterans Affairs (VA) collected data on 852 veterans who screened positive for PTSD.
The following table shows some of their results. You may consider these a random sample of all veterans diagnosed with PTSD.

<table>
<thead>
<tr>
<th>Served</th>
<th>Received Minimally Adequate Treatment</th>
<th>Did Not Receive Minimally Adequate Treatment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>115</td>
<td>280</td>
<td>395</td>
</tr>
<tr>
<td>Elsewhere</td>
<td>165</td>
<td>292</td>
<td>457</td>
</tr>
<tr>
<td>Total</td>
<td>280</td>
<td>572</td>
<td>852</td>
</tr>
</tbody>
</table>


a. If you select one of these 852 veterans at random, what is the probability that he or she served in Iraq or Afghanistan? What is the probability that he or she served in Iraq or Afghanistan given that he or she received minimally adequate treatment?

b. According to the mathematical definition of independent events, are the events served in Iraq or Afghanistan and received minimally adequate treatment independent in this sample? Explain what your conclusion means in practical terms.

c. Using the marginal totals in the table above, complete a table of expected frequencies for a chi-square test of independence. Round to the nearest whole number.

d. Compare the expected frequencies and the observed frequencies. Do the differences seem relatively large or small?

e. Compute the chi-square statistic $\chi^2$.

f. To what critical value should your value of $\chi^2$ be compared? Is the value of $\chi^2$ statistically significant?

g. Write a conclusion that can be drawn from your analysis.
SUMMARIZE THE MATHEMATICS

Reports in the media often contrast two groups or discuss the association between two conditions.

a. What is the difference between absolute risk reduction and relative risk?

b. What are the characteristics of a well-designed experiment? Why is it imperative to conduct an experiment rather than rely on anecdotal evidence when deciding how well a treatment, medical or otherwise, works?

c. Describe the importance of each of the following in an experiment: control group, placebo, single blind, double blind.

d. What does it mean for two groups to be homogeneous? What does the stacked bar graph (percent on the vertical axis) look like if the groups are homogeneous?

e. In a test of homogeneity, what does $\chi^2$ measure? How do you use proportional reasoning to compute expected frequencies?

f. Suppose you have two random samples, each classified into the same categories. How can you tell whether the difference in the proportions that fall into each category is statistically significant? What does it mean when the difference is statistically significant?

g. What statistics can be used to evaluate the effectiveness of a screening test? Which is the one you would want to know if you tested positive for some condition? Which is the one you would want to know if you tested negative?

h. What does it mean if two categorical variables are independent? How can you use the definition of independence to compute expected frequencies for a chi-square test of independence?

i. Describe the similarities and differences between a chi-square test of homogeneity and a chi-square test of independence.

Be prepared to share your ideas and reasoning with the class.

CHECK YOUR UNDERSTANDING

Write, in outline form, a summary of the important statistical concepts and methods developed in this unit. Organize your summary so that it can be used as a quick reference in your future work.